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Progress in Risk Measurement[1]

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Abstract: In this paper, we give the axiomatic characterization of risk measures and discuss the treads of developments in this area. The main recently proposed risk measures are presented, and their properties and relations are discussed. The corresponding versions of dynamic risk measure are also briefly introduced.

Keywords: Coherent risk measures; Value-at-Risk; Expected shortfall

1 Introduction

The term risk plays a pervasive role in the literature on economic, political, social and technological issues. There are various attempts to define and to characterize the risk for descriptive as well as prescriptive purpose (e.g., Brachinger and Weber [16] surveyed measures of perceived risk, regarding risk as a primitive). In this paper, we mainly concentrate on financial risks and financial risk measures. We assume that financial risks can be quantified on the basis of a random variable X. This random variable may present, for instance the future net worth of a position, the relative or absolute changes in values of an investment or the accumulated claims over a period for a collective of insureds. In general, we regard risk as random profit or loss of a position. It can be positive (gains) as well as negative (losses). Pure loss situations can be analyzed by considering the random variable L=$X^-\geq 0$. A risk measure is a mapping from the random variables representing risks to the real line. It gives a simple number that quantifies the risk exposure in a way that is meaningful for the problem at hand.

Financial institutions, such as investment banks and insurance companies around the globe are searching for techniques to enhance their risk management practice.

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One of the critical steps is to construct a proper measure of risk. Possibly enhanced by the new trends in regulation of financial institutions and the reaction of the academic community to the practical requirements, risk measurement is one of the fast evolving topics both in theoretical and practical fields in recent years.

The rest of paper is organized as follows. Section 2 is about new research developments. Section 3 and section 4 are both about static risk measures. One contains the axiomatic characterization of risk measures and the other discusses several typical examples. Section 5 describes dynamic risk measures. And section 6 briefly concludes.

2 New research development

There has been a great momentum in research on risk measures in recent years, which has touched the following different, but interconnected aspects: 1) axiomatic characterization of risk measures; 2) construction of (coherent) risk measures; 3) premium principles in insurance context; 4) dynamic risk measures; 5) the relation between risk measures and other economic and financial theories. 6) application of risk measures to financial activities.

The first line of research was started by a group of scholar: Artzner, Delbaen, Eber, and Heath. The axiomatic definition of coherent risk measures was introduced in their path-breaking paper [8, 9]. They prescribed what mathematical properties a meaningful risk, more precisely capital requirement measures should have. Föllmer et al. [35] and Frittelli et al. [38] proposed convex risk measures respectively. Basing on deviation measures, Rockafellar et al. [59] put forward expected-bounded risk measures. Their results will be presented in the next section of this paper.

In the pre-Markowitz era, financial risk was considered as a correcting factor of expected return. These primitive measures had the advantages of allowing an immediate preferential order of all investments. Variance was first proposed by Markowitz to measure the risk associated with the return of assets. Value-at-Risk (VaR) was introduced in 1994 by the leading bank—JP Morgan. It is very popular in practice and has become part of financial regulations (Basel Committee on Banking Supervision [13, 14]). Conditional Value-at-Risk (CVaR) has been proposed as a natural remedy for the deficiencies of VaR, which is not a coherent risk measure in general. Many other kinds of risk measures are also constructed after the introduction of coherent risk measures. We will give some representative examples later.

Independently, practically at the same time, Wang, Young and Panjer presented similar conclusions on a closely related subject related to insurance premium. Wang et al. [67] published the axioms for risk premium in a competitive insurance market. The most recent results of this line can be seen in Landsman and Sherris [49].

Dynamic risk measures have also been studied for evaluating multi-period risks. In real situations, risks are inherently multi-period due to intermediate cash flows caused by, for example, availability of extraneous cash, possible gains or losses because of the changes of economic situations or adjustment of a portfolio. Having to account for intermediate cash flows is the essential difference between a single-period risk measure and a multi-period risk measure. Furthermore, dynamic risk measures are also need to be considered for intermediate monitoring by supervisors and assessing the risk of a position over time. Föllmer and Leukert [33], and Cvitanić et al. [23] have studied dynamic risk measures. Artzner et al. [11], Riedel [56], and Balbás [12] are all trying to define coherent risk measures for dynamic models. We shall present some results in the section 5 of this paper.

Risk measures relate closely to utility function and asset pricing theories. Based on the classical $(\mu, \sigma)-$portfolio optimization theory of Markowitz (μ and σ refer to expected return and standard deviation respectively), $(\mu, \rho)-$portfolio optimization where ρ is coherent risk measure can be considered. The $(\mu, \rho)-$problem can be transformed into the problem of maximizing U=$\mu - \lambda\rho$, where λ is a Lagrangian variable. U can be interpreted as a utility function just when ρ is convex. On the other hand, when the preference function $\Phi(X) = E[U(X)]$, where U denotes the utility function specific to each decision maker, do not separate risk or value, Jia and Dyer [43] proved that it is possible to derive an explicit risk measures: $\rho(X) = -E[U(X - E(X))]$ for the specified utility function. Jaschke and Küchler [42] proved that coherent risk measures are essentially equivalent to generalized arbitrage bounds, named "good deal bounds" by Černý and Hodges [21]. Thus coherent risk measures link well with the established economic theories of arbitrage on one hand and utility maximization on the other hand. Delbaen [26] has related the theory of risk measures to game theory and distorted probability measures. We can see Wang [67], and Carr [20] for other aspects in this line.

Risk measures have been widely applied to pricing, hedging, portfolio optimization, capital allocation and performance evaluation. To some extent, we can say that the application to performance measurement and capital allocation are the driving force to develop the risk theory. Since the end of 1996 in the European Union and 1998 in the United States, the largest banks subjected to regulatory approval have been able to use the internal models to calculate VaR exposures for the trading books, and thus the capital requirements for market risk to prevent insolvency. After coherent risk measures have been proposed, how to allocate risk capital by selecting proper risk measures become an important issue. When the risk measure for a portfolio has been chosen, how to attribute risk contributions to subportfolio is another problem arising. This is of interest for risk diagnostics of a portfolio or for performance analysis. See Tasche [63], Denault [28], Goovaerts et al. [39] and Fischer [32] for further results. Other aspects of application can be found in Wang [67], Föllmer and Leukert [33].

3 Axiomatic characterization of risk measures

3.1 Coherent risk measures

Following the system of Artzner et al., we regard risks as future net worth. Denote Ω the set of states of of nature, and assume it is finite; $\mathcal{G}$ the set of all risks, namely the set of all real valued function on Ω. For simplicity, we will consider the market models without interest rates in this section; it is immediate, however, to extend all the definitions and results to the situation with interest rate, by appropriately discounting.

Definition 3.1 *A map $\rho : X \to \mathbb{R}$ will be called a coherent risk measure if it satisfies the following axioms:*

(a) Subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y), \quad \forall X, Y \in \mathcal{G}$;

(b) Positive homogeneity: if $\lambda \geq 0$, then $\rho(\lambda X) = \lambda\rho(X)$;

(c) Monotonicity: if $X \leq Y$, then $\rho(X) \geq \rho(Y)$;

(d) Translation invariance: if $m \in \mathbb{R}$, then $\rho(X + m) = \rho(X) - m$.

Let us make some comments on the economic significance of these axioms.

Subadditivity (a): It has an easy interpretation. If the subadditivity did not hold, then $\rho(X + Y) \geq \rho(X) + \rho(Y)$. This would imply, for instance, that in order to decrease risk, a firm might be motivated to break up into different incorporated affiliates. From the regulatory point of view, this would allow to reduce capital requirements. Note this axiom rules out the "semi-variance" type risk measure defined by $\rho(X) = -E(X) + \sigma^2(X - E(X))^-$.

Positive homogeneity (b): We notice that subadditivity implies $\rho(\lambda X) \leq \lambda\rho(X)$ for $\lambda \geq 0$ and $X \in \mathcal{G}$. Thus $\rho(\lambda X) \geq \lambda\rho(X)$ is imposed by the positive homogeneity axioms. This can be justified by liquidity considerations: a position (λX) could be less liquid, and therefore more risky, than that of λ smaller positions (X).

Monotonicity (c): It is obvious to expect that, if two final net worth are such that $X \leq Y$, their risk measures have to satisfy $\rho(X) \geq \rho(Y)$. This axiom rules out the risk measure defined by $\rho(X) = -E(X) + \alpha \cdot \sigma(X)$, where $\alpha > 0$.

Translation invariance (d): Implies that the risk $\rho(X)$ decrease by m, by adding a sure return m to a position X. Specially, we get

$$\rho(X + \rho(X)) = \rho(X) - \rho(X) = 0,$$

that is, when adding $\rho(X)$ to the initial position X, we obtain a "neutral" position.

Typically, a coherent risk measure ρ can be represented by the supremum of the expected negative of final net worth for some collection of "generalized scenarios" or probability measures $\mathcal{P}$ on states of the nature, i.e.,

$$\rho(X) = \sup_{p \in \mathcal{P}} E_p[-X]$$

The axioms of coherent risk measures have been very influential. These coherent risk measures can be used as (extra) capital requirements to regulate the risk assumed by market participants, traders, and insurance underwriters, as well as to allocate existing capitals. But we should realize that not all coherent risk measures are reasonable to use under certain practical situations.

Coherent risk measures were extended in general spaces by Delbaen [26]. Later were extended to convex risk measures, also called weakly coherent risk measures by relaxing the constraints of subadditivity and positive homogeneity, and instead requiring the following weaker condition:

Convexity (e): $\rho(\lambda X + (1-\lambda)Y) \leq \lambda\rho(X) + (1-\lambda)\rho(Y), \quad \forall \lambda \in [0,1]$.

Definition 3.2 *A map $\rho : X \to \mathbb{R}$ will be called a convex risk measure if it satisfies the condition of convexity (e), monotonicity (c), and translation invariance (d).*

Under the assumption $\rho(0) = 0$, Convexity of ρ implies that

$$\rho(\lambda X) \leq \lambda(X), \quad \forall \lambda \in [0,1], \quad \forall X \in \mathcal{G},$$

$$\rho(\lambda X) \leq \lambda(X), \quad \forall \lambda \geq 1, \quad \forall X \in \mathcal{G},$$

The second inequality suggests that when λ becomes large, the whole position (λX) is less liquid than λ singular position X. The first says when λ is small, the opposite inequality must hold for certain reasons.

Convex risk measures take into account the situations where the risk of a position increase in a nonlinear way with the size of the position. They assure for second order stochastic dominance and has corresponding structure theorem:

$$\rho(X) = \sup_{P \in \mathcal{P}} \big(E_P[-X] - \alpha(P)\big),$$

where $\alpha : \mathcal{P} \to (-\infty,\ \infty]$ satisfy $\alpha(P) > -\rho(0)$ for any $P \in \mathcal{P}$, and can be taken to be convex and lower semicontinuous on $\mathcal{P}$.

3.2 Expectation-bounded risk measures

We introduce deviation measures first.

Definition 3.3 *A map $\rho : X \to \mathbb{R}$ satisfying subadditivity (a), positive homogeneity (b), and the following two axioms are called deviation measures:*

(f) Shift-invariance: $\rho(X+m) = \rho(X), \quad \forall X \in \mathcal{G}, m \in \mathbb{R}$;

(g) Nonegative: $\rho(X) \geq 0, \quad \forall X \in \mathcal{G}$.

We can see that $\rho(X) > 0$ for all nonconstant X, whereas $\rho(X) = 0$ for all constant X. Shift-invariance implies deviation measures is location-free. They fully measure the degree of uncertainty of X. Standard deviation and semi-standard deviation are typical examples of this kind. Deviation measures and coherent risk measures are in fact mutually incompatible: there is no function can satisfy axioms (e) and (f) at the same time. The two kinds measure risk from different points of view: the former regard risk as the magnitude of deviation, however, the latter quantify risk to determine the amount of capital has to be held as a cushion against potential future losses.

Rockafellar et al. impose the conditions (a), (b), (d) and the following additional condition:

(h) $\rho(X) > E(-X)$ for all nonconstant X, and $\rho(X) = E(-X)$ for constant X.

Risk measures satisfying the above four conditions are called expectation-bounded. If monotonicity is further satisfied, we will have an expectation-bounded coherent risk measures. The basic ideal of Rockafeller et al. is that applying a risk measure in Artzner et al. sense not to X itself, but to $X - E(X)$ will induce a deviation measure and vice versa. Coherent risk measure and deviation measure are therefore connected together. Formally there is a one-to-one correspondence between expectation-bounded risk measure and deviation risk measures. A simple examples for this correspondence is $\rho_1(X) = b \cdot \sigma(X)$ for $b > 0$ and $\rho_2(X) = b \cdot \sigma(X) - E(X)$. There exist risk measures both coherent and expectation-bounded. Take $\rho(X) = -E_P(X) + \alpha \cdot \sigma_P^-(X)$ with $0 < \alpha < 1, \sigma_P^-(X) = \Big(E\big(\max(E(X) - X, 0 \big)^2 \Big)^{1/2}$ for example.

4 Some examples of risk measures

Variance and standard deviation have been traditional risk measures in economics and finance since the pioneering work of Markowitz. The two risk measures exhibit a number of nice technical properties. For example, the variance of a portfolio return is the sum of the variance and covariance of the individual returns. Furthermore, variance can be used as a standard optimization function. Finally, there is a well established statistical methods to estimate variance and covariance. However, variance does not account for fat tails of the underlying distribution and therefore is inappropriate to describe the risk of low probability events, such as default risks. Secondly, variance penalizes ups and downs equally. Moreover, mean-variance decisions are usually not consistent with the expected utility approach, unless returns are normally distributed or a quadratic utility function is chosen.

A general class of downside risk is the class of lower-partial-moment of degree $k (k = 0, 1, 2, \cdots)$:

$$LPM_k(c, X) = E\big[\max(c - X, 0)^k \big],$$

or, in normalized form $(k \geq 2)$: $LPM_k(c, X)^{1/k}$, where c denote a reference level from which deviation are measured. It can be an arbitrary deterministic target or even a stochastic benchmark.

Let $k = 1$, we get expected regret:

$$ER(X) = E\big[\max(c - X, 0)\big],$$

It was utilized by Carino and Ziemba in the Russell Yasuda Kasai financial planning model. It is actually an average portfolio underperformance compared to a fixed target or some benchmark portfolio. ER can be computed by a linear programming model based on scenario approach.

Let $c = E(X), k = 1$, we have lower-semi-absolute deviation :

$$E\big[\max(E(X) - X, 0)\big],$$

which is considered by Ogryczak and Ruszczynski [52] and Gotoh and Konno [40].

Let $c = E(X), k = 2$, we get semi-variance and semi-standard deviation.

In the following, I shall consider some other familiar risk measures: VaR, CVaR, expected shortfall (ES), and spectral risk measures in detail.

4.1 Value-at-Risk

VaR is a very easy and intuitive concept. It points out how much one may lose during specified period (e.g. two weeks) with a given probability and how much capital should be set to control the risk exposure of a firm. VaR serves for the determination of the capital requirements that banks have to fulfill in order to back their trading activities.

We join here Delbaen [27] taking VaR^{α} as the absolute value of the worst loss not to be exceeded with a probability of at least $1 - \alpha$.

Define

$$x_{(\alpha)} = q_{\alpha}(X) = \inf\big\{x \in \mathbb{R} : P[X \leq x] \geq \alpha\big\}$$

as the lower $\alpha-$ quantile of X,

$$x^{(\alpha)} = q^{\alpha}(X) = \inf\big\{x \in \mathbb{R} : P[X \leq x] > \alpha\big\}$$

as the upper $\alpha-$ quantile of X.

Its formal definition is:

$$VaR^{\alpha}(X) = -x^{(\alpha)} = q_{1-\alpha}(-X).$$

Let $\alpha \in (0,\ 1]$ be fixed. Consider the risk measure ρ given by

$$\rho(X) = VaR_{\alpha}(X),$$

Then ρ has the following properties:

(1) Monotonicity: if $X \leq Y$, then $\rho(X) \geq \rho(Y)$;

(2) Positive homogeneity: $\rho(hX) = h\rho(X), \quad \text{for } h \geq 0$;

(3) Translation invariance: $\rho(X + a) = \rho(X) - a, \quad \text{for } a \in \mathbb{R}$;

(4) Law invariance: if $P[X \leq t] = P[Y \leq t]$ for all $t \in \mathbb{R}$, then $\rho(X) = \rho(Y)$;

(5) Comonotonic additivity: $\rho(f \circ X + g \circ X) = \rho(f \circ X) + \rho(g \circ X), \quad$ for f, g non-decreasing.

Law invariance is a crucial condition for a risk measure to be estimated from empirical data and ensures that the measures are suitable for industrial applications. So it is a critical property for application. There have been many methods to estimate VaR (see Duffie and Pan [30] for an overview). Examples are historical simulation methods and J.P. Morgan's RiskMetrics [46].

From shareholders' or managements' perspective, the quantile "VaR" at the company level is a meaningful risk measure since the default event itself is of primary concern, and the size of shortfall is only secondary. But VaR in general turns out to be not even a convex measure and in particular not subadditive even when the two random variables are independent. This is its major drawback. The subadditivity plays a fundamental role in risk measurement, especially in the area of credit risk. Only in the very special case in which the joint distribution of return is elliptic, VaR is subadditive, i.e.,

$$VaR^{\alpha}(X + Y) \leq VaR^{\alpha}(X) + VaR^{\alpha}(Y), \quad X, Y \in \mathcal{G}$$

Non-subadditivity means that the risk of a portfolio may be larger than the sum of stand-along risks of its components. Thereupon it could happen that a well diversified portfolio require more regulatory capital than a less diversified portfolio. Hence, managing risk by VaR may fail to stimulate diversification and it prevents to add up to the VaR of different risk sources.

VaR is a risk measure that only concerns about the frequency of defaults, not the size of defaults. it is argued that VaR is an "all or nothing" risk measure. VaR models are usually based on the assumption of normal asset returns and will not work under extreme price fluctuations. If an extreme event causes ruin occurs, there is no more capital to cushion losses. Basak and Shapiro found that having embedded VaR into an optimization framework, VaR risk managers incur large losses than non-risk managers in the most adverse states of the world.

Furthermore, it is inappropriate to use VaR in practice because of its non-convexity. It can have many local extremes, which will lead to unstable risk ranking. VaR is a model dependent risk measurement because, by definition, it depends on the initial reference probability.

At the latest in 1999, when coherent risk measures appeared, it became clear that VaR cannot be considered as an adequate risk measurement to allocate economic capital for financial institutions. In spite of this, as a compact representation of risk level, VaR has met the favor of regulatory agencies to measure downside risk and has been embraced by corporate risk managers as an important tool in overall risk management process.

4.2 Expected shortfall

The term ES we used here stems from Acerbi et al. [2], despite the fact that in other literatures this term was already used sometimes in other meanings. Rockafellar and Uryasev [57] have given the formal definitions of $CVaR^+$, $CVaR^-$, and CVaR. They distinguished the three clearly. In fact, CVaR in their paper is the same with ER here. Expected shortfall is proposed as a remedy for the deficiencies of VaR and is characterized as the smallest coherent and law invariant risk measurement to dominate VaR.

In simple words, ES at a specified level α is the average loss in the worst $100\alpha\%$ cases. It measures how much one can lose on averages in states beyond the VaR^α level. Assume $E[X^-] < \infty$, The accurate definition of ES_α is:

$$ES_\alpha(X) = -\alpha^{-1}\Big(E[X\mathbf{1}_{\{X\le x_{(\alpha)}\}} + x_{(\alpha)}\big(\alpha - P[X \le x_{(\alpha)}]\big)\Big), \tag{4.1}$$

where $\mathbf{1}$ is the indicator function

$$\mathbf{1}_A(a) = \begin{cases} 1, & a \in A, \\ 0, & a \notin A. \end{cases}$$

When the underlying distribution is continuous, (4.1) will become simple:

$$ES_\alpha(X) = \alpha^{-1}E\big[X\mathbf{1}_{\{X\le x_{(\alpha)}\}}\big].$$

We can prove that ES is a coherent risk measure. Further more,

$$ES_\alpha(X) = -\alpha^{-1}\int_0^\alpha q_u(X)du. \tag{4.2}$$

Together with the properties of VaR, (4.2) implies that ES is law invariant and comonotonic additive. It also shows ES is the coherent risk measure used in Kusuoka [48] as main building block for the representation of law invariant coherent risk measurements.

ES_α is continuous with respect to α. Hence regardless of the underlying distributions, we can be sure that the risk measured by ES_α will not change dramatically when there is a switch in the confidence level, say some base points.

ES is monotonic function to α, that is

$$ES_{\alpha+\varepsilon}(X) \le ES_\alpha(X), \quad \forall\alpha \in (0,1), \;\; \forall\varepsilon > 0, \; with \;\; \alpha + \varepsilon < 1.$$

Another advantages of ES is that it can be estimated efficiently even in cases where the usual estimators for VaR fail. First define $\alpha-$tail mean (TM) as:

$$TM_\alpha(X) = -ES_\alpha = \alpha^{-1}E\big[X\mathbf{1}_{\{X\le x_{(\alpha)}\}}\big] = \bar{x}_{(\alpha)}.$$

Then TM_α is the average the worst $100\alpha\%$ cases. The tail mean is likely to be negative, and the ES represents potential loss as positive number in most cases. Indeed they are different names for the same object.

Let some independent samples $(X_1,\ X_2,\ \cdots, X_n)$ of X are given. Define the order statistics $X_{1:n} \le \cdots \le X_{n:n}$ as the sorted values of n-tuple $(X_1,\ X_2, \cdots, X_n)$. Approximate the number of $100\alpha\%$ elements in the sample by

$$[n\alpha] = \max\left\{\ m \ \middle|\ m \le n\alpha, \quad m \in \mathbb{N}\right\}.$$

The set of $100\alpha\%$ worst cases is therefore presented by the least $[n\alpha]$ outcomes $\left\{X_{1:n}, \cdots, X_{[n\alpha]:n}\right\}$. The natural estimator for the expected loss in the $100\alpha\%$ worst cases is therefore simply given by

$$TM_\alpha^n(X) = \frac{\sum_{i=1}^{[n\alpha]} X_{i:n}}{[n\alpha]}.$$

We can get

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{[n\alpha]} X_{i:n}}{[n\alpha]} = \bar{x}_{(\alpha)} \qquad a.s. \tag{4.3}$$

If X is integrable, then the convergence in (4.3) holds in L_1 too.

More importantly, ES is a convex function with respect to positions, allowing the construction of efficient optimizing algorithms. In particular, it has been shown that ES can be minimizing using linear programming techniques, which makes many large-scale calculations practical. And moreover, such numerical calculation is efficient and stable.

ES is different from the tail conditional expectation (TCE) and worst conditional expectation (WCE) advanced by Artzner et al. [7]:

$$TCE^\alpha = -E\left\{X \middle| X \le x^{(\alpha)}\right\},$$

$$WCE_\alpha(X) = -\inf\left\{E[X|A]:\ A \in \mathcal{F},\ P(A) > \alpha\right\}.$$

TCE^α is usually larger than the set of selected worst cases since $\{X \le x^{(\alpha)}\}$ may happen to have a probability larger than $100\alpha\%$. It is a coherent risk measure only when restricted to continuous distributions while may violate subadditivity on general distributions. The natural estimation for TCE is the negative of average of all $X_i \le x_n^{(\alpha)}$, *i.e.*,

$$TCE_n^\alpha(X) = -\frac{\sum_{i=1}^{n} X_i \mathbf{1}_{\{X_i \le X_{[n\alpha]:n}\}}}{\sum_{i=1}^{n} \mathbf{1}_{\{X_i \le X_{[n\alpha]:n}\}}}.$$

WCE is a coherent risk measure but only useful in a theoretical setting since it depends not only on the distribution of X but also on the structure of the underlying probability space. Both TCE and WCE are sensible to small changes in the confidence level α when applied to discontinuous distributions.

Comparing ES, TCE and WCE, we get

$$TCE^{\alpha}(X) \leq WCE_{\alpha}(X) \leq ES_{\alpha}(X).$$

ES is the maximum of WCEs when the underlying probability space varies. If the distribution of X is continuous, we have

$$TCE^{\alpha}(X) = WCE_{\alpha}(X) = ES_{\alpha}(X).$$

ES closely relates to expected return (ER). Testuri and Uryasev [65] demonstrated the relationship between ER and ES. They formally prove that a portfolio, which minimizes ES, can be obtained by doing a sensitivity analysis with respect to the threshold in the expected regret. An optimal portfolio in ES sense is also optimal in the expected regret sense for some threshold in the regret function. The inverse statement is also valid, i.e., if a portfolio minimizes the expected regret, this portfolio can be found by doing a sensitivity analysis with respect to the ES confidence level. A portfolio, optimal in expected regret sense, is also optimal in ES sense for some confidence level.

When comparing expected shortfall with VaR, basing on Monte-Carlo simulation, Yamai and Yoshiba [73] get that expected shortfall is easily decomposed into risk factors and optimized, while VaR is not, but expected shortfall requires a larger size of sample than VaR for the same level of accuracy. Risk decomposition enables risk managers to select assets that provide the best risk-return trade-off, and to allocate economic capital to individual risk factors. Under market stress, Yamai and Yoshiba [72] find that: first, VaR and expected shortfall may underestimate the risk of securities with fat-tailed properties and a high potential for large losses; second, VaR and expected shortfall may both disregard the tail dependence of asset returns; third, expected shortfall has less of a problem in disregarding the fat tails and the tail dependence than VaR does. The superintendent office of financial institutions in Canada has put in regulation for the use of expected shortfall to determine the capital requirement.

4.3 Spectral risk measure

Spectral risk measures $M_{\phi}(X)$ are defined by

$$M_{\phi}(X) = -\int_0^1 x_{(p)}\phi(p)dp, \tag{4.4}$$

where $\phi \in \mathcal{L}^1([0,1])$. And ϕ is said to be an admissible risk spectrum if it is positive, decreasing and $\|\phi\| = 1$.

$M_\phi(X)$ is a coherent risk measure if and only if ϕ is an admissible risk spectrum. ϕ can be regarded as a weight function reflecting an investor's subjective risk aversion. This means there exists a one-to-one correspondence between risk aversion function ϕ and coherent spectral risk measures. The fact that $\phi(p)$ is decreasing provides an intuitive insight of the coherence concept: a measure is coherent if it assigns bigger weight to worse cases. Introduce a measure $d\mu(\alpha)$ on $\alpha \in [0, 1]$ satisfying the normalization condition

$$\int_0^1 \alpha d\mu(\alpha) = 1.$$

We can get

$$M_\phi(X) = -\int_0^1 x_{(p)}\phi(p)dp = \int_0^1 ES_{(\alpha)}(X)d\mu(\alpha), \tag{4.5}$$

Eq. (4.5) suggests spectral risk measure can be built by ES_α.

Let $\phi(p) = \frac{1}{\alpha}\mathbf{1}_{\{0\leq p\leq\alpha\}}$, Then

$$ES_\alpha(X) = M_\phi(X).$$

In this case, The weight function is simply uniform in $p \in [0, \alpha]$ and zero elsewhere. In general case, $\phi(p)$ assigns different weights to different "p-quantile slices" of the left tail.

It is not difficult to see that $VaR^\alpha(X)$ can also be expressed as

$$VaR^\alpha(X) = M_\phi(X) \quad with \quad \phi(p) = \delta(p - \alpha),$$

where Dirac delta function is defined by

$$\int_a^b f(x)\delta(x - c)dx = f(c), \quad \forall c \in (a, b)$$

Spectral risk measure enjoys the properties of law-invariance and comonotonic additivity. It is also a very simple object to be used in practice. We can estimate M_ϕ by M_ϕ^n on a sample n $i.i.d$ realizations $X_1, \cdots, X_n$ of X. $X_{i;n}$ has the same meaning as before. Let

$$M_\phi^n(X) = -\sum_{i=1}^n X_{i:n}\phi_i,$$

where ϕ_i satisfies: $(1)\phi_i \geq 0;\ (2)\phi_i \geq \phi_j,\ if\ i < j;\ and\ (3) \sum\limits_{i=1}^n \phi_i = 1.$

Given $\phi(p)$ as a representative positive decreasing function with $\sup\limits_{p\in[0,1]} \phi(p) < \infty$, the most natural choice when ϕ_i is

$$\phi_i = \frac{\phi(i/n)}{\sum\limits_{k=1}^n \phi(k/n)}, \quad i = 1, \cdots, n$$

If $E[X^+] < \infty$ *and* $E[X^-] < \infty$, we can prove that M_ϕ^n is a consistent estimator: converges to M_ϕ with probability 1 for $n \to \infty$.

From the perspective of portfolio optimization, Acerbi et al. [1] revealed that risk-reward problem is shown to coincide with the unconstrained optimization of a single suitable spectral measure. In other words, minimizing a spectral measure turns out to be already an optimization process itself, where risk minimization and returns maximization cannot be disentangled from each other.

Spectral risk measures have some similarities with distorted risk measures defined on distorted risk probabilities:

$$\rho(X) = E^*(X) = -\int_{-\infty}^{0} g(F(x)dx + \int_{0}^{\infty} [1 - g(F(x)]dx,$$

where $g : [0,1] \to [0,1]$ is an increasing function with $g(0) = 0$ and $g(1) = 1$. Distortion function g can reflect the risk preference of an investor. Proper risk measures can be got by correctly choosing function g.

There are numerous risk measures. In the above, we just illustrate some typical examples.

5 Dynamic risk measure

Consider a finite time interval $[0, T]$ during which all economic activities take place, and suppose that the target is a liability C with maturity T. In a complete market under the no arbitrage assumption, consider an agent who cannot afford to commit at time $t = 0$ the entire amount

$$C(0) = E\Big[\frac{C}{S_0(T)}\Big],$$

which would guarantee perfect hedging. Here $S_0(t)$ is the price of risk-free instrument at time t in the market. The expectation E is calculated with respect to the unique, risk-neutral equivalent martingale measure. Then C is a risk that has to be properly measured.

Cvitanić and Karatzas [23] proposed measuring risk as the smallest expected discounted net-loss:

$$\rho(x, C) = \inf_{\pi(\cdot)\in\mathcal{A}(x)} E_0\left(\frac{C - X^{x,\pi}(T)}{S_0(T)}\right)^+, \tag{5.1}$$

where x is the initial capital available at time $t = 0$; $\mathcal{A}(x)$ is the class of admissible portfolio strategies; $X^{x,\pi}(\cdot)$is the wealth process corresponding to portfolio $\pi(\cdot)$ and initial capital x. Under certain conditions, explicit computation are got and it is then straightforward to determine the smallest amount of capital that keeps the exposure to risk below a given, acceptable level. This is the dynamic version of the static lower-partial-moment risk proposed to amend the dynamic VaR version:

$$\rho(x, C) = \sup_{\pi(\cdot)\in\mathcal{A}(x)} P_0\big[X^{x,\pi}(T) \geq C\big].$$

which fails to take into account the magnitude of the net hedging loss $\big(C - X^{x,\pi}(T)\big)^+$. (5.1) can be extended to the following form:

$$\rho(x, C) = \inf_{\pi(\cdot)\in\mathcal{A}(x)} E_0\Big[l\Big(\frac{C - X^{x,\pi}(T)}{S_0(T)}\Big)^+\Big],$$

where l is a concave function.

It is possible that the agent faces some uncertainty of the financial market itself in addition to genuine risk that the liability C represents. Under this situation, we capture the uncertainty by allowing for a suitable family $\mathcal{P} = \{P_\nu\}_{\nu\in\mathcal{D}}$ of "real world probability" or "scenarios" that are general enough to incorporate the uncertainty about the actual values of stock-appreciation rates, instead of just one (P_0). Possible measures are set to control the risk amount, which is bounded by a lower-measure of risk:

$$\rho(x, C) = \sup_{\nu\in\mathcal{D}} \inf_{\pi(\cdot)\in\mathcal{A}(x)} E_\nu\left(\frac{C - X^{x,\pi}(T)}{S_0(T)}\right)^+, \tag{5.2}$$

which corresponds to the maximal risk of the type (5.1) from the point of view of an agent faced with the "worst possible scenario" $\nu \in \mathcal{D}$; an upper-measure of risk:

$$\rho(x, C) = \inf_{\pi(\cdot)\in\mathcal{A}(x)} \sup_{\nu\in\mathcal{D}} E_\nu\left(\frac{C - X^{x,\pi}(T)}{S_0(T)}\right)^+, \tag{5.3}$$

which is viewed by a regulator who regards the agent's efforts merely as attempts to "contain the worst that can happen". The quantity of (5.2) and (5.3) can be thought of as the lower (max-min) and upper (min-max) values of fictitious "stochastic game between the agent and the market respectively. A saddle point of this game is shown to be the pair $(\hat{\pi}(\cdot), \hat{\nu})$, where $\hat{\pi}(\cdot)$ corresponds to the investment strategy which borrows the amount $C(0) - x$ from the bank at time zero, and then invests in the market according to the optimal hedging portfolio $\pi_C(\cdot)$ for C, $\hat{\nu}$ is the risk-neutral measures included in the possible "real world" measures. Coincidence of (5.2) and (5.3) means that the agent and regulator can reach agreement about how the risk associated with the liability C is to be quantified.

In a stable hedging with finite probability space, $x = 0, S(T) = 1$, and $\mathcal{A}(0)$ only consisting of $\pi(\cdot) = 0$, we have

$$\rho(C) = \rho(0, C) = \sup_{\nu\in\mathcal{D}} E_\nu(C^+).$$

The quantity of (5.2) satisfies the following properties:

(1) $\rho(x, C) \leq \|(C - x)^+\|_\infty = ess\sup(C(\omega) - x)^+$;

(2) $\rho(x_1 + x_2, C_1 + C_2) \leq \rho(x_1, C_1) + \rho(x_2, C_2)$;

(3) $\rho(\lambda x, \lambda C) = \lambda\rho(x, C)$, for $\lambda \geq 0$;

(4) $x \longmapsto \rho(x, C)$ is convex increasing,

$x \longmapsto x + \rho(x, C)$ is convex increasing for fixed C.

These properties are related to single-period coherent risk measures.

Siu et al. [61] proposes Bayesian risk measures for derivatives which is easy to be implemented and satisfies the four axioms of coherent risk measures. Their approach provides investors with more flexibility in measuring risks of derivatives, by using Gerber Shiu's option-pricing formula. The newly introduced concept—Bayesian Esscher scenarios, takes both the subjective views and the market observation into account.

It is more complex and difficult to consider dynamic risk measures. The corresponding systematic axioms that dynamic risk measures should satisfy have been tried to propose by lots of scholars. Each set of axiomatic characterizations has its own reasonable part. But none becomes so influential as single-period coherent risk measures. Multi-period models are still the subject of ongoing research. The existing ones characterize dynamic risk measures from different angle. Here I just illustrate two to bring some heuristics for further research.

Wang [70] proposed likelihood-based risk measures and gave the corresponding structure form. This kind of risk measures satisfies six properties: continuity, risk separability, consistency, stationarity, future independence and timing indifference. But Wang dees not assume translation invariance. Therefore the corresponding class of risk measures need not to be coherent nor convex.

Riedel [56] considered the changes of a position and availability of new information. Changes in the position are to be taken into account by recalculating the (stochastic) present value of future payments. Information is processed via updating in a Bayesian way every single probability measure in the set of generalized scenarios. He assumed that a dynamic measures should satisfy the following conditions: independence of the past, adaptedness, monotone and predictable translation invariance. In addition to these four conditions, a coherent risk measure should be homogenous and subadditivity. Riedel showed that every dynamic risk measure that satisfies the axioms of coherence, relevance and dynamic consistence can be represented as the maximal expected present value of future losses where expectations are taken with respect to a set of probability measures.

6 Conclusion

The importance of a risk measure is in its ability to differentiate between different types of risk, its ability to accurately and consistently compare the severity of different risk portfolio, and its ability to be easily understood and applied. VaR does not take into account the severity of of an incurred damage event and fails to stimulate diversification. It will lead to disastrous results when used to measure risk in most financial situations. As a response to these deficiencies, Artzner et al. first introduced the notion of coherent risk measures and put forward the axioms a reasonable risk measure has to satisfy. This set of axioms has been widely accepted and regarded as a landmark in the field of risk theory. Since then, researches on the basis of coherent risk measures have been carried on. Convex and expectation-bounded risk measures were put forward. Expected shortfall is proposed as a natural coherent alternative to VaR. At the same time, it enjoys the properties of continuity and monotonicity in the confidence level. Moreover, it has efficient estimator and the corresponding portfolio optimization problem can be solved by linear programming methods. Other different classes of (coherent) risk measures with their own properties are also constructed.

There also arise lots of related problems, for example, the relationship between coherent risk measures and other financial theories and how to choose proper risk measures in practice when we have so many choices. While optimizing a portfolio or allocating risk capital by companies as well as by regulators, we will want to known which is the best: variance, VaR or some other coherent risk measures, the relationship of efficient frontiers obtained by solving the portfolio selection problem under these measures or the principle of allocating capital among portfolios. Goovaerts et al. [39] examine properties of risk measures that can be considered to be in line with some "best practice" rules in insurance, and argue that so-called coherent risk measures lead to problems. Financial risk include market risk, credit risk, operation risk and so on. Each kind of risk has its own distinctive characters. Furthermore, newer financial instruments surface in the capital market every day. It is still great challenges to properly and effectively measure these risks. Compared to single-period risk measures, dynamic risk measures are more complicate and difficult. The corresponding axiomatic characterizations are still an ongoing research. On the fast developing topic of risk measurement, much work has already been done, however, more further researches still need to be carried on.

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î€World Scientiï¬c Publishing Company. Progress in risk management. Guest editorsâ€™ introduction. Kin keung lai.Â Complexities involved in risk measurement and management are growing at least. as fast as, if not faster than, tools and methodologies available to those actually. responsible for managing risk in real life situations, as well as academic researchers, implying that risk management continues to be an under-researched subject. Having. agreed on this basic fact, the guest editors determined that the purpose of this. special issue should be â€œto assess the current state of knowledge about measurement. and management of risk and to generate and throw open for discussion, more ideas 6. Risk measurement and market dynamics. 7. Stress testing and financial stability policies. Part 1 opening remarks, concluding remarks and dinner address.Â Bank credit risk, common factors, and interdependence of credit risk in money markets. Abstract. 8. Irrespective of the risk management and risk measurement practices adopted, a bankâ€™s operational risk strategy should reflect the nature and source of the bankâ€™s operational risks for all Operational Risk Measurement System (ORMS) elements, including regular review of predictive elements against experience.Â (a) Overall, banks have made considerable progress in the collection and use of internal loss data since the previous international LDCE, conducted in 2002; (b) The frequency of internal losses of â‚¬20,000 or more varies significantly across regions when the data are scaled by various exposure indicators; (d) Despite the regional variation in loss frequency noted above, there is some consistency in the severity distribution of operational losses across regions Monetary and Capital Markets. Exchange Rate Risk Measurement and Management: Issues and Approaches for Firms. WP/06/255. Prepared by Michael Papaioannou1.Â Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate. Measuring and managing exchange rate risk exposure is important for reducing a firmâ€™s vulnerabilities from major exchange rate movements, which could adversely affect profit margins and the value of assets. Risk assessment is all about measuring and prioritizing risks so that risk levels are managed within defined tolerance thresholds without being overcontrolled or forgoing desirable opportunities. Events that may trigger risk assessment include the initial establishment of an ERM program, a periodic refresh, the start of a new project, a merger, acquisition, or divestiture, or a major restructuring. Some risks are dynamic and require continual ongoing monitoring and assessment, such as certain market and production risks.Â Most organizations define scales for rating risks in terms of impact, likelihood, and other dimensions. These scales comprise rating levels and definitions that foster consistent interpretation and application by different constituencies.