Gödel's Impact on Hilbert's Problems for Cost Estimating
or
Cost Consistency and Completeness as an Impossible Exercise

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In two previous papers, presented to the 2007 joint conference of the Society of Cost Estimating and Analysis (SCEA) and the International Society of Parametric Analysts (ISPA); and subsequently a revision presented at the International Cost Estimating and Analysis Association (ICEAA), Braxton and Coleman used Hilbert's Problems as catalyst to propose (2007) and revisit (2013) their formulation of Hilbert's Problems for Cost Estimating. What follows in this paper employs Gödel's theorems w/respect to Hilbert's application onto the cost estimating community. What can we learn about ourselves as estimators and where can we exert greatest impact with our estimates? Using Gödel's two theorems of undecidability as catalyst, we explore the effect and utility of exacting mathematics and other notions on cost estimates specifically and programmaitics generally.

German mathematician David Hilbert put forth a list of 23 mathematics problems[1] that were unsolved at the time. His influential presentation, "The Problems of Mathematics," was given on 8 August 1900 in the Sorbonne at the Paris conference of the International Congress of Mathematicians. As the most renowned mathematician of his time, "Hilbert believed that everything in mathematics could and should be proved from basic axioms."[2] Thus, enabling an answer to every single question and freeing the discipline of inconsistencies. Hilbert's famous talk influenced the course of mathematical activity for the next century.

He selected the 23 problems he felt most important and some related to the general field of mathematics. The problems were aimed to serve as examples of the kinds of problems whose solutions would lead to advancing mathematics. Therefore, some were areas for investigation and not strictly problems for calculated solutions. Most of the problems focused on the logical structure of the discipline, and many were not new. Immediate results occurred with respect to

[1] Hilbert originally had 24 problems on his list, but decided against including one of them in the published list. The "24th problem" was rediscovered in Hilbert's original manuscript notes by German historian Rüdiger Thiele in 2000.
[2] Singh, Simon. Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem. Anchor Books: New York, N.Y.; 1997, page 136.

some of the problems, while others remain unsolved today. The abundance of success is partially attributable to Hilbert's statement of the problem and definition/explanation, as some of the 23 problems had existed for hundreds of years. Those problems not quickly resolved still benefited from Hilbert's program via conversation and debate, some even sparking the creation of mathematical subdisciplines.

Hilbert retired in 1930 confident that mathematics was on a restorative, unifying path. He anticipated a consistent logic sufficient to answer every mathematical question. Then in 1931, Kurt Gödel, aged 25, burst upon the scene, publishing a paper that forever devastated Hilbert's vision of a definitive and consistent logic of mathematics.[3] Thus, Gödel joined Hilbert is one of the two greatest figures of twentieth century mathematics. In an interesting side note, although Hilbert lived for 12 years after Gödel's theorem, no indication has been found that he wrote any response to Gödel's work.

Informally, Gödel's first incompleteness theorem states that all consistent axiomatic formulations of number theory include undecidable propositions.[4] Gödel's second incompleteness theorem states that if number theory is consistent, then a proof of this fact does not exist. Gödel had proved a complete and consistent mathematical system was impossible. The exact opposite of Hilbert's dream. The importance of Gödel's first incompleteness theorem is that it provides a negative answer to Hilbert's program, asking whether mathematics is "complete," in the sense that every statement in number theory can be either proved or disproved. In his second incompleteness theorem, Gödel proves that all consistent formulations include undecidable propositions.[5] Simply put: the first incompleteness theorem shows, "If axiomatic set theory is consistent, there exist theorems that can neither be proved or disproved" and the second incompleteness theorem strengthens the first in that, "There is no constructive procedure that will prove axiomatic theory to be consistent."[6] So:

[3] Hofstadter, Douglas R. Metamagical Themas: Questing for the Essence of Mind and Pattern. Basic Books, Inc.: New York, N.Y.; 1985, page 485.
[4] Hofstadter, Douglas R. Gödel, Escher, Bach: An Eternal Golden Braid. Vintage Books: New York, N.Y.; 1980, page 17.
[5] Ibid, page 17.
[6] Singh, Simon. Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem. Anchor Books: New York, N.Y.; 1997, page 139.

> Essentially Gödel's first statement said that no matter what set of axioms were being used there would be questions that mathematics could not answer — completeness could never be achieved. Worse still, the second statement said that mathematicians could never even be sure that their choice of axioms would not lead to a contradiction — consistency could never be proved.[7]

Well, you might be asking, 'What does this mean for cost estimating? … What is your point?' Please, stay with me a bit longer… We're getting there.

The application of Gödel's incompleteness theorems to fields other than logic and mathematics can provide broader revelations. However, strict application or sound reasoning can be questionable, especially if the system being studied is not sufficiently axiomatic. Some have tried to apply Gödel's conclusions "in contexts where its relevance is at best a matter of analogy or metaphor."[8] These systems are not formal in the logic sense – examples include philosophy, quantum mechanics, the Bible, and the legal system, where no formal rules of inference exist.

One influential application of Gödel was by Roger Penrose.[9] He argues that human brains cannot be given a full explanation in terms of currently understood physics because there's just something about a human mathematician that can somehow *see* the consistency of a *formal system* – like the analytic truth of axioms – which ought to be prevented by Gödel's theorem, if our brains were just formal systems in the sense of machines. Hence, Penrose rejects the plausibility of strong artificial intelligence, pending the discovery of something like quantum gravitational effects in the human brain. Hofstadter made another interesting assertion of a Gödelian loop "which limits the depth to which any individual can penetrate into his own psyche? …is it not reasonable to expect that we cannot mirror our complete mental structures in the symbols which carry them out?"[10]

Almost there…

In their 2007 paper, Braxton and Coleman presented a sequel to Hilbert's problems, a set of problems for cost estimating, and an award winning update in 2013. I affectionately think of these as the Braxton-Coleman Problems. Like Hilbert's, the Braxton-Coleman problems range in topic

[7] Ibid, page 141.
[8] Franzén, Torkel. "The Popular Impact of Gödel's Incompleteness Theorem." Bulletin of the American Mathematical Society, Volume 53, Number 4 (April 2006), page 440.
[9] Penrose, Roger and Gardner, Martin. The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics. Oxford University Press: New York, N.Y.; 1989.e
[10] Hofstadter, Douglas R. Gödel, Escher, Bach: An Eternal Golden Braid. Vintage Books: New York, N.Y.; 1980, page 697.

and precision. They did a great job organizing the problems facing the field of cost estimating into four categories: professional identity; analytical techniques; cost estimating implementation; and integration with other disciplines. Also like Hilbert, they embrace new developments and specialties within our discipline. The Braxton-Coleman program should be of intense interest to cost estimators appreciative of how it all fits together, as well as those doing research with respect to the specific problems clarified and defined. Braxton and Coleman have admirably done for cost estimating what Hilbert did for mathematical logic. These problems should receive continued study until resolved; and may well guide cost estimation research for the coming decades. However, care must be taken not to construe the Braxton-Coleman program into a complete unified theory of cost estimating with internal consistency. I do not believe that is their intent, but the casual reader might leap to an implicit or explicit conclusion that once all these problems are solved the cost estimating community we will have arrived at an axiomatically consistent system wherein our construction is provable.

The reason for such a warning stems from the abundance of research we costers do to show many things with extreme rigor and advanced mathematics. As Hilbert pointed out:

> This conviction of the solvability of every mathematical problem is a powerful incentive to the worker. We hear within us the perpetual call: There is the problem. Seek its solution. You can find it by pure reason, for in mathematics there is no *ignorabimus*.[11]

Many cost conferences contain rigorous mathematics seeking to show or prove esoteric points related to cost estimation. Often these interesting and analytically stimulating problems have limited applicability to the field of practice. Whereas, practitioners of cost estimating have firm mathematics backgrounds and enjoy the search for solutions, we must remember that cost estimating is an inexact science.

However, unlike the above examples of religious texts, philosophy, and the like that apply Gödel's conclusions as metaphor and analogy, cost estimating uses a lot of mathematics and formal rigor to demonstrate and apply pure science to what inherently has social science aspects, albeit significantly influential ones. If we fail to remain conscious of the limitations under which we

[11] Hilbert, David. "Mathematical Problems." Bulletin of the American Mathematical Society, Volume 8, Number 10; 1902, pp. 437-479. The Latin maxim *ignoramus et ignorabimus*, meaning "we do not know and will not know," stood for a position on the limits of scientific knowledge, in the thought of the nineteenth century.

strive, the incentive mentioned by Hilbert has the potential to blind us to Gödel's devastating effect on a vision of a consistent logic to answer all cost estimating questions. The effort is a noble one; and the caution is just that, a reminder to be judicious in our claims, as cost consistency and completeness is an impossible exercise.

We must always remember that cost estimating is as much art as it is science.[12] No matter how rigorous the math we extract and apply, the totality of our programmatic estimates, there context, and there decision environment are influenced by people that serve as inputs to the problems. Thus, we must be equally mindful of the social science side of our business. The solution isn't confined to the equations; it's also influenced by human biases. Further, the overall complexity is vast, requiring principles from other disciplines, in a widening array of possibilities beyond our set of problems. All this, taken together, might help form a consensus understanding of the problem, its contextualization, and lead to generally accepted solutions or applications. What we learn makes the search worthwhile, but a finite answer or set of answers is unobtainable. One of the main goals of Hilbert's program was a finitistic proof of the consistency of the axioms of arithmetic; this shouldn't be a goal of the cost estimating field. While it might take us decades to resolve specific Braxton-Coleman problems, informed by Gödel, we should have a better understanding of what we're working toward.

Gödel's incompleteness theorems are among the most important results in modern logic. His discoveries revolutionized the understanding of mathematics and logic, and have potentially dramatic implications for fields of study heavily reliant on mathematics. Using Gödel's two theorems of undecidability as catalyst, the effect and utility of exacting mathematics was explored as it relates to problems in cost estimating. Examining the impact of Gödel on Hilbert's problems for cost estimating, the limitations of completeness and consistency were shown. We learned that estimators should focus on more than the mathematical problems at hand and impact estimates, as well as decision makers in a broader social sense. The Braxton-Coleman program is very valuable to the cost estimating community. Future updates are highly desired and invaluable to progress within the discipline of cost estimating.

[12] Peeler, Jr., David L. "The Art of Costing: Musings on Estimating, War, & Analytical Rigor." *National Estimator*, Spring 2011.

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Hilbert's problems are twenty-three problems in mathematics published by German mathematician David Hilbert in 1900. They were all unsolved at the time, and several proved to be very influential for 20th-century mathematics. Hilbert presented ten of the problems (1, 2, 6, 7, 8, 13, 16, 19, 21, and 22) at the Paris conference of the International Congress of Mathematicians, speaking on August 8 at the Sorbonne. The complete list of 23 problems was published later, most notably in English translation in In mathematics, Hilbert's second problem was posed by David Hilbert in 1900 as one of his 23 problems. It asks for a proof that the arithmetic is consistent â€“ free of any internal contradictions. Hilbert stated that the axioms he considered for arithmetic were the ones given in Hilbert (1900), which include a second order completeness axiom. In the 1930s, Kurt GÃ¶del and Gerhard Gentzen proved results that cast new light on the problem. Some feel that GÃ¶del's theorems give a negative solution to the Is it known how Hilbert initially reacted to GÃ¶del's incompleteness theorems upon their announcement at the KÃ¶nigsberg conference in 1930, or their publication in 1931? mathematics mathematicians biographical-details mathematical-logic hilbert. ï»¿ Share.Â somewhat angry but then began to try to deal constructively with the problem Broadened methods would permit the loosening of the requirements of formalizing. Hilbert himself now took a step in this direction. This was the replacing of the schema of complete induction by a stronger rule called "transfinite induction".Â GÃ¶del himself gave a consistency proof for Peano arithmetic using so-called functionals of higher types (see Shoenfeld's Mathematical Logic). ï»¿ Share. Improve this answer.