The Deflected Beam in Differential Calculus: Bringing engineering into the mathematics classroom

G.V. Loganathan[1], Bill Greenberg[2], Lorraine Holub[2], Craig Moore[1]

[1]Departments of Civil and Environmental Engineering /
[2]Department of Mathematics,
Virginia Tech, Blacksburg, VA 24061

Introduction

First year calculus is a hard course for the freshmen students and is sometimes considered as a weed out class by the students. However, as Malcom and Triesman[1] convincingly argue, the calculus course is the pump for all other classes and should not be treated as a filter. In addition, the time varying/hidden cognitive abilities of beginning college students require proper nurturing to come to fruition, especially through cooperative learning and guided instruction (Malcom and Triesman[1]; Wankat and Oreovicz[2]; Felder and Brent[3]). The two emerging crucial considerations are: (1) calculus must be learned as a foundation for all modern science and engineering, and (2) it is not that the students lack ability, it is that their talents are quite different than earlier generations and, consequently, misplaced priorities interfere with the didactic presentation. While the first commitment is the strategic goal, the second one requires formulation of action items to achieve the goal.

Curriculum Change and Instructional Methods

Universities all over the country have embarked on various plans for better teaching of calculus. These may be grouped into three categories: (1) introduction of innovative instructional methods/aids, (2) reordering and in general minor additions and deletions of topics to serve a wider class of students, and (3) integration of mathematics, physics and chemistry with focus on a particular field such as engineering. Categories 2 and 3 deal with alteration of the syllabus whereas category 1 focuses mainly on instructional methods.

Barrow and Fulling[4] and SimCalc (http://www.simcalc.umassd.edu/simcalc/curriculum) curriculum are two good examples for category 2 content modification. Barrow and Fulling argue for introducing vectors and line integrals in the first calculus course along with the derivatives. Simcalc curriculum supports accumulation and integrals before rates and derivatives. The IMPEC (Integrated Mathematics, Physics, Engineering and Chemistry) curriculum (Felder et al.[5]) points out the compartmentalized nature of the science, mathematics, and engineering courses. In this regard Townsend et al.[6] have also noted the impracticality of introducing Engineering/Science word problems and explaining them in a short period. In the IMPEC curriculum elements of engineering design and operations are brought into the first year and integrated with the calculus and science courses. While the IMPEC curriculum focuses on multidisciplinary integration (with emphasis on engineering), the overall course contents with the main sections (non IMPEC) of chemistry, physics, and calculus are preserved.

In category 1, the content is not altered but various instructional methods/aids are used. These include: *word problems* which require students to identify the variables and express them in algebraic symbols, identify the process rules (constraints) in the problem and write them in terms of the symbols, solve the problem and interpret the results; *interactive mathematical packages* capable of animation to display results as conditions are altered; *mathematical writing* (http://www.fandm.edy/departments/mathematics) emphasizing clear explanations, uncluttered expositions on the page, and well organized presentation; *tutorials* involving a significant number of drill problems with help on demand for students working in peer groups or on an individual basis so that mistakes are fixed without any delay. A component that should be added to the above list is a well-conceived set of hands-on experiments. It is well known that what you remember the most is what you have learned by doing. While that statement is readily accepted, the time constraint combined with the multidisciplinary nature of experiments (mini projects) is often cited for not carrying it out as part of calculus class. At Virginia Tech, the Emerging Scholars Program (ESP) includes all of the aforementioned components as part of the calculus curriculum.

The Emerging Scholars Program (ESP)

Although the concept of Supplemental Instruction as an enhancer of student success dates back over 90 years to John Dewey, the model for the current Emerging Scholars Programs (ESP) in the Mathematics Department of Virginia Tech dates from Uri Treisman's collaborative workshops with underachieving minorities at Berkeley in the 1980's. The value of such programs at Berkeley, Texas, California-Davis, Wisconsin, and (since fall 1996) Virginia Tech in increasing the rate of student success in such traditionally difficult courses as freshman calculus has been widely documented. The goals of the Virginia Tech ESP project (as stated in the final report on the 1996-97 pilot project) are as follows:

- To increase the number of students who succeed in the engineering calculus sequence
- To increase the understanding of mathematical concepts and knowledge of mathematical skills
- To provide enrichment opportunities for academically successful math students, by bringing together faculty, undergraduate teaching assistants and calculus students in a supportive learning environment.
- To provide a framework for learning and academic success within a rigorous and challenging calculus sequence.
- To offer interactive experiences for mathematically talented students which can serve as a mechanism for retaining departmental majors.

The Emerging Scholars Program Class

The two courses in the first year engineering/math/physical science calculus sequence are the three credit Math 1205 differential calculus and Math 1206 integral calculus courses.

Approximately 50% of enrolled students are required to take the one credit ESP class supplementing each of these lecture classes (See the next section for selection process). The ESP class meets twice a week for seventy-five minutes at a time. Each ESP class is a group problem-solving class in which the students work together in small groups on problem sets that the ESP teachers put together. The final grade for the one-credit ESP course of A or F (and occasionally, C in marginal cases) is based on attendance and active participation. Each section has about 40 students in it and is taught by a faculty member or experienced graduate teaching assistant. In addition, each section has three undergraduate teaching assistants. Within this structure there is certain amount of variation, e.g. using overheads for student presentations, working in pairs, trios or larger groups, and the like. There is a strong dialogue between the three-credit teacher of Math 1205 and the one-credit ESP teacher. The problems that the one-credit class work on are designed to help the students enhance the concepts, supplement the experience, correct the difficulties, etc. that the students have in the three-hour class. Although the class is not designed to be remedial, the weak backgrounds in basic algebra and trigonometry skills exhibited by many of the students are addressed in the context of solving calculus problems. An indirect benefit from its structure is that it imparts discipline and good study habits at a critical period in an adolescent’s life. It should be emphasized that the lecture sections of Math 1205/1206 which have attached ESP classes and those which do not both use the same syllabus and are administered the same common final examination.

Student Selection

For each student entering Virginia Tech, a *mathematics readiness* score is derived from a regression model which includes high school GPA, math SAT score, and whether the student has had calculus in high school. The score has been an excellent predictor of student success in the beginning mathematics sequences. All students enrolling in the Math 1205/1206 sequence who fall below a fixed threshold in mathematics readiness score are required to take the supplementary ESP course. For fall Semester 1998 there are seventeen sections of Math 1205 with ESP sections and seventeen sections of 1205 which do not have ESP classes. Many of the students in the ESP sections would have previously started in a presently defunct precalculus course. The students were dispirited to be in there, were getting behind in their math courses which were prerequisites for engineering courses, and did not understand why they needed yet another precalculus course, since they felt they had had the material in high school. Statistics showed strongly that the precalculus class was very ineffective, since only a small percentage of the students who started in it finished the engineering calculus sequence.

Performance

The first year that the ESP program was implemented in Math 1205 on a large scale was fall 1997, so the following statistics are from the 1997-98 school year. At the outset, the difference between the two groups of students (non-ESP and ESP) was considerable: almost 19 points in the mean math readiness score and almost 60 points in the mean math SAT score. Such gaps would ordinarily translate to differences in mean course grades of about a letter (B versus C, e.g.) and in mean common final exam scores of several correct answers (out of 17 questions in all). Indeed, with a failure rate in MA 1205 exceeding 15% and in Math 1206 exceeding 20% in years prior to 1997, and with all predicted at-risk students in ESP sections, it might be expected that the failure rate in the ESP sections would exceed 25% and 30% respectively. Here's what actually happened:

	Math Readiness Mean	Math SAT Mean	1205 Mean Grade	1205 Exam Mean # Correct	1205 Grade of F percent
Non-ESP	87.7	653.3	2.7	9.3	4.6%
ESP	68.8	594.6	2.4	7.8	6.5%

As would be expected the non-ESP students outperformed the ESP students in both categories. However, the actual differences in 1205 course grades (B- versus C+) and common exam results (1.5 correct responses) clearly indicate that the ESP approach is quantitatively beneficial for student outcomes.

Even more impressive are the results compiled from the Math 1206 classes the following semester, spring 98, which show what statistics at other schools have shown, that the gap between the ESP and non-ESP students narrows as the ESP students progress through the year of the program. This is amazing in that typically, especially for weak students, grades go down from Math 1205 to Math 1206. But here's what actually happened:

	Math Readiness Mean	Math SAT Mean	1206 Mean Grade	1206 Exam Mean # Correct	1206 Grade of F percent
Non-ESP	86.4	665	2.39	8.93	11.7%
ESP	69.3	620	2.17	8.79	12.4%

The non-ESP students, as expected, again outperformed the ESP students in both categories, but the difference in mean 1206 course grades between non-ESP and ESP students was 0.18 --- about one-sixth of a letter grade. The difference in mean number of correct answers on the common exam was 0.14 --- about one-seventh of a correct answer. This agrees with experiences at Berkeley, UC-Davis and other schools: namely, that by the end of the second semester of calculus, the ESP students have practically caught up with the other students. All of this is a clear indication that the ESP approach is quantitatively beneficial for student outcomes. Preliminary indications are that the same types of results will occur again this year. A new component that is being added to two sections of the ESP project is a set of hands-on experiments.

Hands-on Experiments

A widely held belief is that the mathematics, physics, chemistry and engineering courses are highly compartmentalized and integration is needed. The motivation for including a few hands-on experiments as part of the ESP project stems from the observation (Malcom and Treisman) "Where the relationship between a science experiment or a design problem and the mathematics is made clear, students seem to perform better and are more highly motivated. Too often the trend has gone in the wrong direction – not only a separation of mathematics from the hands-on activities… but also a substitution of mathematics for hands-on experience and practical understanding…”. Being truthful to the statement by Malcom and Treisman, a set of hands-on laboratory experiments has been formulated.

It is envisaged to have a maximum of 3 experiments (3 weeks) in a 15-week semester. Currently, a structural mechanics experiment (Beam Bending), a hydraulics experiment (flow over a weir) and an electrical engineering experiment (RC-circuit) have been formulated. The selection of an experiment depends on how well it can blend calculus and engineering concepts and not to have too many mathematical ideas that fall outside the scope of the current calculus class. For experiments related to MA1205 there should not be any analyses containing integration or differential equations but only concepts involving the derivatives. Also, the experiment should pertain to day-to-day life so that the beginning year students can visualize it immediately. To present the ideas in a concrete manner, the experiment from now on will mean the Beam Bending experiment. Before each lab session, the students are provided with a complete set of notes on the experiment to be performed. The ESP instructors also assign problems emphasizing the current experiment. Each laboratory session entails the following elements.

<u>Motivation</u>

A faculty member presents a 5-minute clear exposition of the application of the experiment (beam) in real life with a set of visual aids. The role of geometrical and material properties is then introduced. With the aid of Figure 1, the question to ask is “Which beam will bend more?”

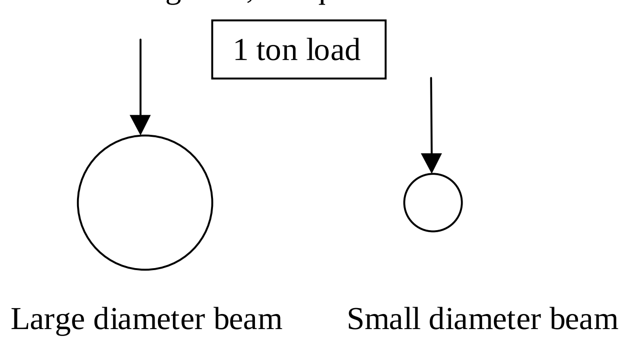


Figure 1. Role of Geometry

After receiving a response from the students, it is revealed that the large diameter beam is made up of rubber and small diameter beam is made up of iron. At this point the role of the material property becomes clear. Also, it should be pointed out how the material property is bundled into a constant called “Modulus of Elasticity”. The term elasticity is defined in terms of the ability of the beam to spring back to its original shape upon removal of the load. At this stage the students are asked to consider a piece of chalk as a beam. How little the chalk deflects under load is emphasized. By breaking the chalk it is pointed out how it fails without warning. Contrasting this behavior with a rubber band, its considerable elongation without breakage under load is noted. The results are interpreted in terms of the “brittleness” and “ductility” properties of materials. The idea of “Area Moment of Inertia” is also introduced. Showing some dramatic **failures** ends the 5-minute motivation lecture**!**

Contents of the Procedural Manual

Each experiment has an accompanying procedural manual. It contains the following sections: (1) *Background*. Examples of real world applications of the experimental object along with the appropriate figures should be included (for a beam it can be a bridge, a massive steel tower and so on). The technical definition of the object and a description of its operating environment including the boundary conditions and mode of failure are provided. [A beam is supported horizontally and subject to bending due to vertical loading. Excessive deflection is not acceptable.] (2) *Clear Objectives*. Using action verbs, what the student should do is clearly stated. Comparing theoretical quantities to their actual measurements derives maximum benefit. Here, innovative measurements should be possible. For example, in beam deflection the slope is evaluated by actually measuring the slope angle along the deflected beam with an "inclinometer" (see Figure 2). The following objectives are stated. By performing this experiment, the student should be able:
(i) to compute the derivative as the slope at a point; (ii) to approximate the derivative as an average rate of change also known as the finite difference approximation; (iii) to calculate the differential. (3) *Calculus background*. Give reference to the exact pages of the textbook in which the related calculus concepts are covered. (4) *Theory*. Present the valid equation from engineering. For beam deflection, point out that when a beam bends due to external loading it develops internal stresses within its cross-section to reach equilibrium. Thus, a relationship between the external loading and cross-sectional and material properties of the beam can be developed. (5) *Experimental Procedure*. Provide a step by step procedure for the entire experiment. (6) *Worksheet*. Provide tables for entering data and analysis. (7) *Questions*. Include questions and problems to extend the concepts and theory learned through the experiment. For the beam experiment, why a hollow rod bends more than the solid rod for the same conditions? Compute the maximum deflection for a diving board when a diver is about to jump. Discuss the effect of end condition.

Analysis of Beam Deflection

Figures 2 and 3 show the deflected beam. A schematic of the deflected beam of Fig.2 alone is presented as Figure 3. The various slope angles are measured with an inclinometer as are the deflections of the beam from a baseline. The depicted variables are defined as part of Eq.(1). The beam deflection, v at distance x from the left end denoted by v(x) is given by

$$v(x) = PaLx/(2EI) - Pax^2/(2EI) \quad \text{for } 0 \le x \le 0.5L \qquad (1)$$

in which: P = load (dyne = $gm.cm/sec^2$), a = equal distance from either end at which P is applied, L = length of beam between the supports(cm), E = material dependent property (modulus of elasticity, $dyne/cm^2$), I = property of cross sectional geometry (moment of inertia, cm^4), v = deflection (cm).

Figure 2. Deflection of Beam

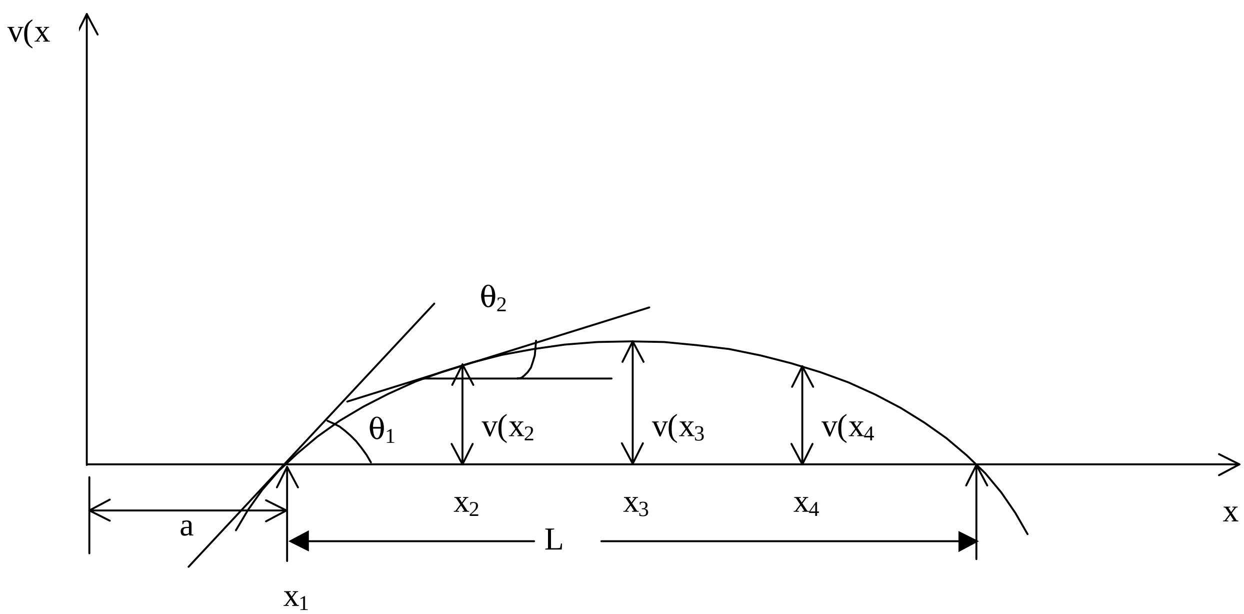


Figure 3. Slope of a curve at a point (deflected beam)

Objective 1. Derivative as the slope

Figures 3 shows the beam with the deflections $v(x_1)$, $v(x_2)$, $v(x_3)$, and $v(x_4)$ at the corresponding points x_1, x_2 , x_3 and x_4. The derivative at point 2 is given by the slope
tan θ_2 measured at x_2 using an inclinometer as

(dv/dx)at 2 = tan θ_2 (2)

Objective 2. Derivative as the average rate of change

The derivative is also approximated by

(dv/dx)at 1 $\cong [v(x_2) – v(x_1)] / (x_2 – x_1) = \Delta v/\Delta x$ at point 1 (3)

which is the average rate of change between points 1 and 2, also known as the finite difference approximation of the derivative.

Equation (1) is differentiated to yield

$(dv/dx)_x = \alpha + \beta x$ for $0 \leq x \leq 0.5L$ (4a)

in which: $\alpha =$ (4b)

$\beta =$ (4c)

The first derivative is differentiated to obtain the second derivative as

$(d^2v/dx^2) =$ for $0 \leq x \leq 0.5L$ (5)

The student is expected to fill in the blank boxes. Equation (5) shows that the second derivative is a constant. The second derivative is very nearly equal to the reciprocal of the radius of the circular arc formed by the deflected beam between its supports. The validity of the statement can be verified by the geometry of Fig.3 and

Sin $\theta_1 = PaL/(2EI)$ (6)

in which: θ_1 = angle measured at the left support. The value of θ_1 obtained from Eq.(6) is compared with its measured value. This verification serves as another means to check the validity of the beam equation (1).

Objective 3. The Differential

Figure 4 shows the role of the differential in approximating the function difference as given below.

The differential $dv = (dv/dx)_{\text{at point 1}}dx$ and $v(x_2) \cong v(x_1) + dv$ (7)

whereas the actual difference in deflections is given by

$$\Delta v = v(x_2) - v(x_1) \qquad \text{OR} \qquad v(x_2) = v(x_1) + \Delta v \tag{8}$$

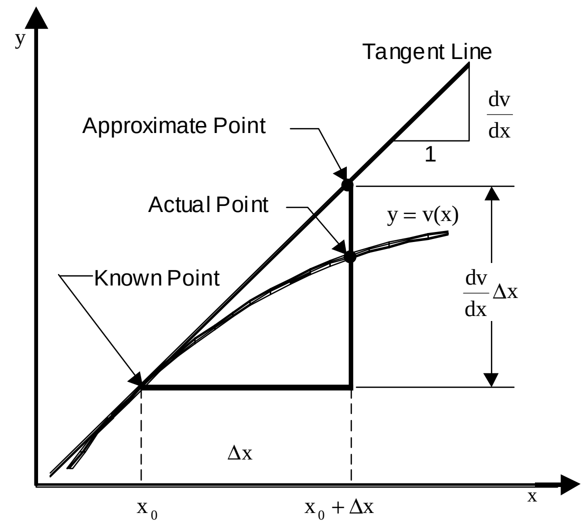


Figure 4. Approximation of Function Difference by the Differential

<u>The Experiment</u>

Place a slender cylindrical rod between two supports. Apply equal loads, P at equal distance, "a" from the supports of the rod (at its extremities) as shown in Figure 2. The beam will deflect upwardly as shown in Figs. 2 and 3. Note down the rod (beam) length between the supports L, load distance a, magnitude of load P, moment of inertia I, and modulus of elasticity E in Table 1. Measure the deflections v(x) at various points x and note them down in Table 2 (To conserve space only three rows are provided). Use the inclinometer to measure the slope angles θ, at various points x. Evaluate tan θ = dv/dx at the same points. Also, use the finite difference approximation, Eq.(3), to obtain Δv/Δx . Compute the differential v′(x) Δx at various points and enter them in Table 2.

Table 1. Properties of Beam

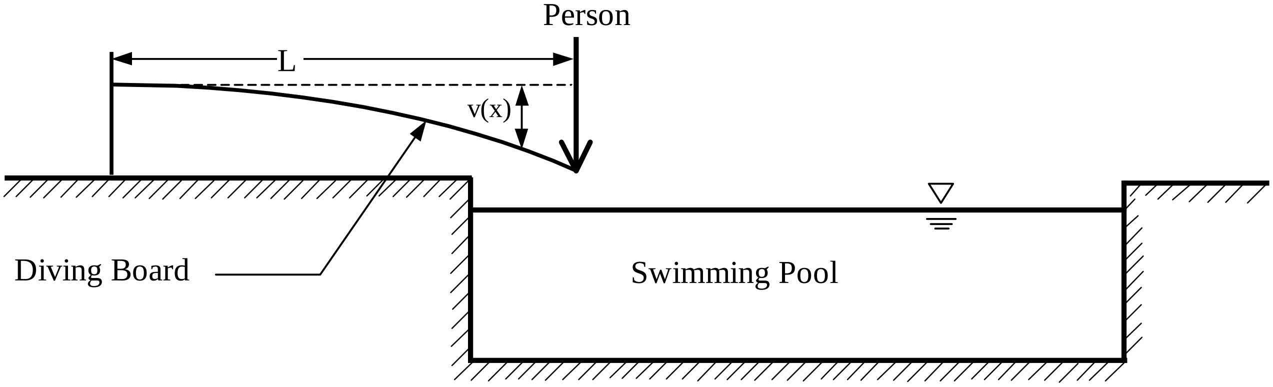
L(cm) =			$I(cm^4) = \pi d^4/4$	
d (cm) =			$E(dyne/cm^2)$ =	
P^*(dyne) = cm/sec^2			α =	
a(cm) =			β(1/cm) =	
1dyne =	$1gm\ cm/sec^2 = 10^{-5}$ Kg m/sec^2 = 10^{-5} Newton			

Table 2 - Deflection Data and Analysis*							
Measured			**Computed**				
1	2	3	4	5	6	7	8
x	θ	$v(x)_{exp}$	$v(x)_{theor}$	$\frac{dv}{dx} \cong \frac{v(x+\Delta x)_{exp} - v(x)_{exp}}{\Delta x}$	$\frac{dv}{dx} = \tan\theta$	$\left.\frac{dv}{dx}\right\|_{Theor}$	$v(x+\Delta x) \approx v(x)_{exp} + \left.\frac{dv}{dx}\right\|_{Theor} \Delta x$
0							
4							
8							

* Table abbreviated

Sample Questions

1. Compare the v_2 values from Eq.(7) with that of Eq.(8). Note that the application of Eq.(7) does not require the exact equation (Eq.1)for deflection.
2. Comment on the differences between dv and Δv.
3. Given a diving board of length, L, modulus of elasticity, E and area moment of inertia, I, and the equation below, compute the *maximum deflection* of the board when a person weighing P is about to jump. Here, $v(x) = \frac{Px^2}{6EI}(3L - x)$ for $0 \le x \le L$



Students' Response

The engineering students seem to be quite enthusiastic about the lab experiment because of its strong engineering content and the depth with which the problem has been presented. They also appreciate the exposure to the various material properties and geometrical properties and their role in deciding the outcome, namely, the deflection. They like getting their hands dirty and seem genuinely interested in taking the readings to the maximum eye ball accuracy possible. Their faces brighten up the minute they see that their measured and computed values agree quite nicely. They have greater appreciation for the engineering equations and, more importantly, the role of calculus which makes it all possible. The minor calculating difficulties include the following. They have a little trouble to treat some symbols as constants. They are inconsistent in treating the accuracy of data. Because of the smallness of the numbers, the CGS system of units are used; however, it leads to minor mistakes in unit conversion in filling out Table 1. It should also be noted that at Virginia Tech, the engineering students take their physics classes only during their sophomore year. Therefore, unless they remember their high school physics well, these minor difficulties are bound to happen.

Discussion

While the hands-on experiments are only a minor component in the overall structure of the ESP project, it is felt that it fills an important void in the way courses are taught. It is also learned that it is not the complexity of the analysis that matters but it is the combination of material and geometrical properties, instrumentation, innovative measurements, exposition of real world problems, and major calamities due to the failure of an important engineering element that matters. Most of all, it is the recognition that without mathematics none of the sciences and engineering will exist. For this experiment, it gives students a physical grounding in the most important —yet forgotten— result of differential calculus: the approximation of the change in functional value by its derivative. Even though the presentation in this paper is focused on the beam deflection problem, the weir flow and the RC circuit problems are of equal interest and provide a broader view of engineering. Also, all three problems lend themselves for an integral analysis covered in MA1206 which is also part of the ESP project. It is the writers' view (also pointed out by Townsend et al.) that unless sufficient time is available as afforded by the ESP project, it is very hard to treat good engineering problems in depth. But the success rate established by the ESP project clearly attests to the fact that it is not the difficulty of the course content (as demonstrated by the poor performance of the students under the remedial precalculus class) but the right support structure that is needed.

Acknowledgment

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Biography

G.V. LOGANATHAN
G.V. Loganathan is an Associate Professor of Civil and Environmental Engineering at Virginia Tech. He and his co-authors are working together to find efficient ways of teaching calculus as part of a project supported by SUCCEED (Southeastern University and College Coalition for Engineering Education) and the Mathematics department. The team is also exploring interesting approaches to teach statistics to civil and environmental engineering students. G.V. Loganathan has a Ph.D. from Purdue University.

WILLIAM GREENBERG
William Greenberg is Professor of Mathematics at Virginia Tech. He received his B.A. from Princeton University and his M.A. and Ph.D. from Harvard University. Prof. Greenberg has conducted theoretical research in statistical mechanics and other areas of mathematical physics for nearly 30 years, and has lectured in more than 25 countries. He has long concentrated on the teaching of undergraduate and graduate mathematics courses to engineers.

LORRIANE HOLUB
Lorraine Holub is an Instructor of Mathematics at Virginia Tech. Having initiated the department's Emerging scholar's Program with a pilot section in 1996, she currently serves as co-coordinator of the now largely expanded program. She received her Bachelor of Science degree and Master of Science degree in Mathematics from Louisiana State University in Baton Rouge, Louisiana.

CRAIG S. MOORE
Craig S. Moore is currently a master's student at Virginia Tech. He received his B.S. degree in Civil Engineering from Virginia Tech. Before returning to graduate school to obtain a degree in Hydrosystems, Craig worked for one year on a construction site for an Army Corps of Engineer's flood control project in Buena Vista, Virginia. In addition to research, Craig works part-time for a local engineering firm and also is developing hands-on labs for a new Civil Engineering course.

Mathematics Teacher Mathematical Knowledge Alternative Conception Mathematics Classroom Mathematical Activity.Â â€˜History in the mathematics classroom: linkages in kinematic geometryâ€™, in H. N. Jahnke, N. Knoche & M. Otte (eds), History of mathematics and education: ideas and experiences, GÃ¶ttingen: Vandenhoeck and Ruprecht, 39â€“67Google Scholar. Bartolini Bussi, M. G. 1998. â€˜Drawing instruments: historical and didactical issuesâ€™, in LR Google Scholar.Â â€˜Historical motivation for a calculus course: Barrowâ€™s theoremâ€™ in R. Calinger (ed.) Vita mathematica: historical research and integration with teaching, Washington: MAA, 309â€“315.Google Scholar. Fourier, J.B. 1822/1955. Integral Calculus. Differential Equations. Advance Engineering Mathematics. Engineering Mechanics. Strength of Materials. Theory of Structures.Â Problem 18 The strength of a rectangular beam is proportional to the breadth and the square of the depth. Find the shape of the largest beam that can be cut from a log of given size. Solution: Click here to show or hide the solution. Dierential calculus is about nding the slope of a tangent to the graph of a function, or equivalently, dierential calculus is about nding the rate of change of one quantity with respect to another quantity. If we are going to go to all this trouble to nd out about the slope of a tangent to a graph, we had better have a good idea of just what a tangent is.Â If you nd that you need to revise this topic you may nd the Mathematics Learning Centre publication Exponents and Logarithms helpful. 3.1 Derivatives of constant functions and powers. Perhaps the simplest functions in mathematics are the constant functions and the func-tions of the form xn. Rule. 1.