

# Theory of Asset Pricing

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# Preface

The genesis of this book comes from my experience teaching asset pricing theory to beginning doctoral students in finance and economics. What I found was that no existing text included all of the major theories and techniques of asset valuation that students studying for a Ph.D. in financial economics should know. While there are many excellent books in this area, none seemed ideal as a stand-alone text for a one-semester first course in theoretical asset pricing. My choice of this book's topics were those that I believe are most valuable to someone at the start of a career in financial research. Probably the two features that most distinguish this book from others are its broad coverage and its user-friendliness.

Contents of this book have been used for over a decade in introductory finance theory courses presented to doctoral students and advanced masters students at the University of Illinois at Urbana-Champaign. The book presumes students have a background in mathematical probability and statistics and that they are familiar with constrained maximization (Lagrange multiplier) problems. A prior course in microeconomics at the graduate or advanced undergraduate level would be helpful preparation for a course based on this book. However, I have found that doctoral students from mathematics, engineering, and the physical sciences who had little prior knowledge of economics often are able to understand the course material.

This book covers theories of asset pricing that are the foundation of current theoretical and empirical research in financial economics. It analyzes models of individual consumption and portfolio choice and their implications for equilibrium asset prices. In addition, contingent claims valuation techniques based on the absence of arbitrage are presented. Most of the consumption-portfolio choice models assume individuals have standard, time-separable expected utility functions, but the book also considers more recent models of utility that are not time separable or that incorporate behavioral biases. Further, while much of the analysis makes standard “perfect markets” assumptions, the book also examines the impact of asymmetric information on trading and asset prices. Many of the later chapters build on earlier ones, and important topics reoccur as models of increasing complexity are introduced to address them. Both discrete-time and continuous-time models are presented in a manner that attempts to be intuitive, easy to follow, and that avoids excessive formalism.

As its title makes clear, this book focuses on theory. While it sometimes contains brief remarks on whether a particular theory has been successful in explaining empirical findings, I expect that doctoral students will have additional exposure to an empirical investments seminar. Some of the material in the book may be skipped if time is limited to a one-semester course. For example, parts of Chapter 7’s coverage of binomial option pricing may be cut if students have seen this material in a masters-level derivatives course. Any or all of the chapters in Section V also may be omitted. In my teaching, I cover Chapter 15 on behavioral finance and asset pricing, in part because current research on this topic is expanding rapidly. However, if reviewer response is any indication, there are strongly held opinions about behavioral finance and asset pricing, and so I suspect some readers will choose to skip this material all together while others may wish to see it expanded.

Typically, I also cover Chapter 16 which outlines some of the important models of asymmetric information that I believe all doctoral students should know. However, many Ph.D. programs may offer a course entirely devoted to this topic, so that this material could be deleted under that circumstance. Chapters 17 and 18 on modeling default-free and defaultable bond prices contain advanced material that I typically do not have time to cover during a single semester. Still, there is a vast amount of research on default-free term structure models and a growing interest in modeling default risk. Thus, in response to reviewers' suggestions, I have included this material because some may find coverage of these topics helpful for their future research. A final note on the end of chapter problems: most of these problems derive from assignments and exams given to my students at the University of Illinois. The solutions are available for instructor download at the Addison Wesley website.

#### *Acknowledgements*

I owe a debt to the individuals who first sparked my interest in financial economics. I was lucky to have been a graduate student at MIT during the early 1980s where I could absorb the insights of great financial economists, including Fischer Black, Stanley Fischer, Robert Merton, Franco Modigliani, Stewart Myers, and Paul Samuelson. Also, I am grateful to my former colleague at Wharton, Alessandro Penati, who first encouraged the writing of this book when we team taught a finance theory course at Università Bocconi during the mid-1990s. He contributed notes on some of the book's beginning chapters.

Many thanks are due to my colleagues and students at the University of Illinois who provided comments and corrections to the manuscript. In addition, I have profited from the valuable suggestions of many individuals from other universities who reviewed drafts of some chapters. I am particularly indebted to the following individuals who provided extensive comments on parts of the book:

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The level of support that I received from the staff at Addison-Wesley greatly exceeded my initial expectations. Writing a book of this scope was a time-consuming process that was made manageable with their valuable assistance. Senior Acquisitions Editor Donna Battista deserves very special thanks for her encouragement and suggestions.

Last but not least my wife Peggy and our triplets George, Laura, and Sally deserve recognition for the love and patience they have shown to me. Their enthusiasm buoyed my spirits and helped bring this project to fruition.



Contingent Claims Theory Leaving aside options, the equity of an investment is the most volatile investment that funds the assets of an economic entity. The equity of an entity controls it and possesses a valuable option - to abandon it all and hand the company over to the next most junior investor. Option valuation can tell us a lot about about the cost of capital. This is my theory of asset/liability/equity pricing in broad. Comments are welcomed. My Theory Of Asset Pricing. Add a Comment. Related Articles. Arbitrage Pricing Theory Formula. In the APT model, an asset's or a portfolio's returns follow a factor intensity structure if the returns could be expressed using this formula:  $r_i = a_i + \hat{\beta}_{i1} * F_1 + \hat{\beta}_{i2} * F_2 + \dots + \hat{\beta}_{in} * F_n + \hat{\mu}_i$ , where  $a_i$  is a constant for the asset;  $F$  is a systematic factor, such as a macroeconomic or company-specific factor;  $\hat{\beta}$  is the sensitivity of the asset or portfolio in relation to the specified factor; and  $\hat{\mu}_i$  is the asset's idiosyncratic random shock with an expected mean of zero, also known as the error term. The APT formula is  $E(r_i) = r_f + \hat{\beta}_{i1} * RP_1 +$  This article compares two leading models of asset pricing: the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT): I argue that while the APT is compatible with the data available for testing theories of asset pricing, the CAPM is not. In reaching this conclusion emphasis is placed on the distinction between the unconditional (relatively incomplete) information which econometricians must use to estimate asset pricing models and the conditional (complete) information which investors use in making the portfolio decisions which determine asset prices. This note introduces asset pricing theory to Ph.D. students in nance. The emphasis is put on dynamic asset pricing models that are built on continuous-time stochastic processes. It is very preliminary. Please let me know if you discover any mistake. Preface. i. 1 Introduction to Asset Pricing Theory. 1. 1.1 Basic Abstractions . . .