Mathematics Curriculum Development and the Role of Problem Solving

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> … a fundamental aim of the mathematics curriculum is to educate students to be active, thinking citizens, interpreting the world mathematically, and using mathematics to help form their predictions and decisions about personal and financial priorities. (NCB, 2009, p. 5).

Problem solving is recognised as an important life skill involving a range of processes including analysing, interpreting, reasoning, predicting, evaluating and reflecting. It is either an overarching goal or a fundamental component of the school mathematics curriculum in many countries. However, developing successful problem solvers is a complex task requiring a range of skills and dispositions (Stacey, 2005). Students need deep mathematical knowledge and general reasoning ability as well as heuristic strategies for solving non-routine problems. It is also necessary to have helpful beliefs and personal attributes for organizing and directing their efforts. Coupled with this, students require good communication skills and the ability to work in cooperative groups (Figure 1).

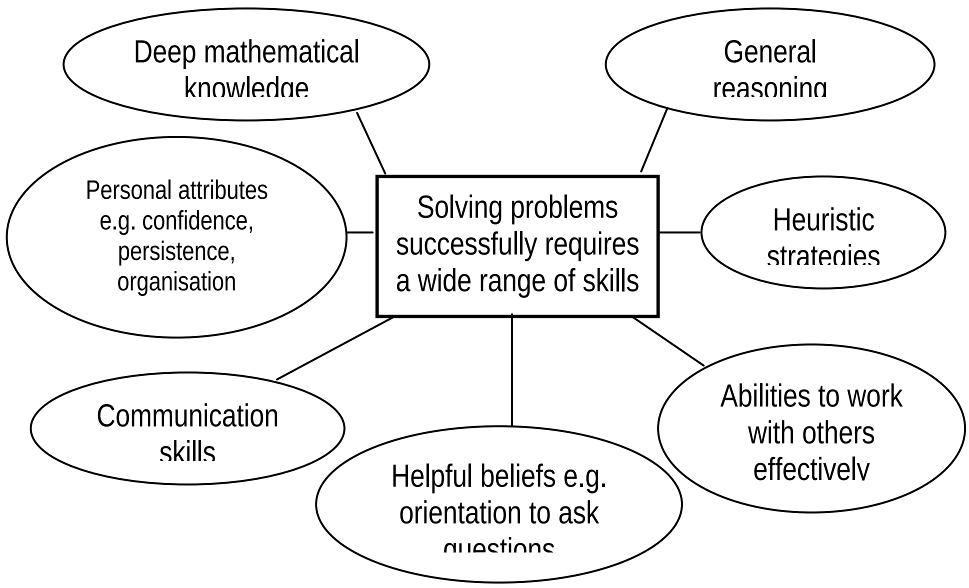

Figure 1. Factors contributing to successful problem solving (Stacey, 2005, p. 342)

Teachers have had many opportunities to build knowledge about teaching problem solving and using problems as a focus of learning in mathematics (Cai, 2003). In Australia advice to teachers has been provided in a range of publications including books (e.g., Lovitt & Clarke, 1988) and professional journals (e.g., Peter-Koop, 2005), in national curriculum statements (e.g., Australian Education Council, 1991) as well as in state and territory curriculum documents (e.g. BOS NSW, 2002). Such advice has been accompanied by pre-service and in-service programs to change teaching practices from more traditional approaches to contemporary or reform methods where teachers use non-routine problems and problem-centred tasks (Anderson & Bobis, 2005). Given the amount of policy advice and resource development, there are concerns about the limited opportunities for Australian students to solve problems other than those of low procedural complexity (Stacey, 2003). It is possible that the main constraints on implementation are the types of questions including in examinations and in textbooks (Doorman et al., 2007; Kaur & Yeap, 2009; Vincent & Stacey, 2008).

As Australia continues the process of developing a national curriculum, it is important to learn from other countries about the best approach for including problem solving in the curriculum and for supporting implementation by teachers. International approaches to supporting teachers are varied with some countries developing realistic tasks (e.g. Holland), and others reducing the content in the curriculum to allow teachers more time for problem solving (e.g. Singapore). Examining the efforts of other countries and considering the constraints and affordances for teaching problem solving will inform the efforts required for successful national curriculum development and implementation in Australia.

International Approaches to Problem Solving in the Curriculum

Many curriculum documents present the school mathematics curriculum as lists of topics or ‘content’ and a set of ‘processes’. Typically content includes the fundamental ideas of mathematics, historically grouped into such topics as number, algebra, measurement, geometry and chance and data. While processes includes the actions associated with using and applying mathematics to solve problems which may be routine or non-routine – in many state and territory mathematics curriculum documents the processes have been grouped together and labelled Working Mathematically (Clarke, Goos & Morony, 2007). The following section summarises the approach to problem solving in the mathematics curriculum and the support provided for teachers in Singapore, Hong Kong, England and the Netherlands – this selection of countries has been chosen to exemplify some of the approaches taken and to highlight issues involved in implementation.

Singapore

In Singapore, the results of an early TIMSS study led to several changes in the curriculum – the content was reduced by about 30% (Kaur, 2001) and problem solving became the primary goal of learning mathematics. Figure 2 represents the framework of the mathematics curriculum with problem solving dependent on five inter-related components – skills, concepts, processes, attitudes and metacognition. The content is presented as skills and processes while attitudes represents the affective dimensions of learning, metacognition highlights the importance of self-regulation, and processes includes acquiring and applying mathematical knowledge.

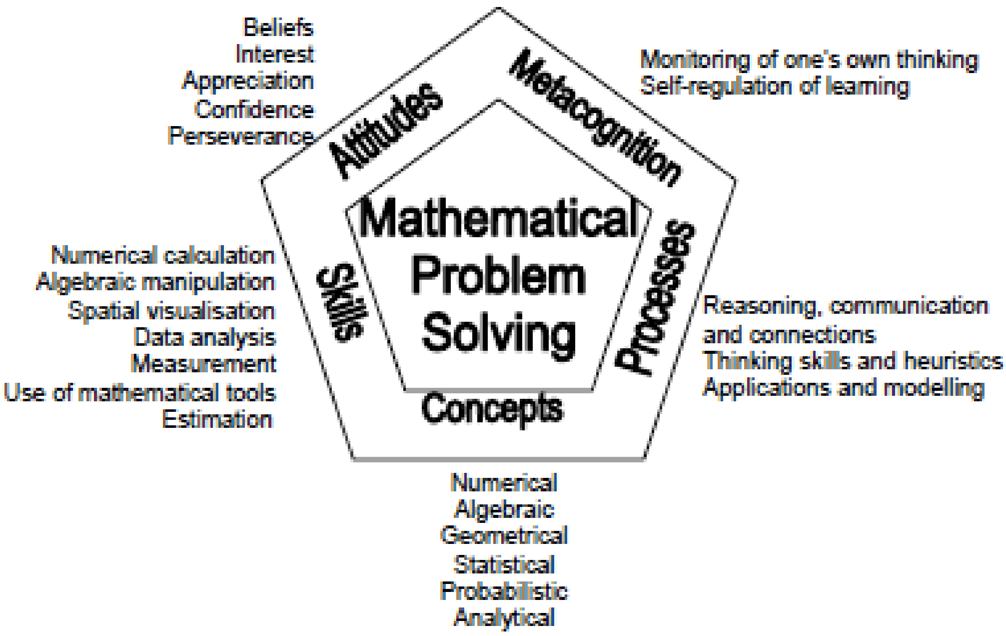

Figure 2. Mathematics framework from the Singapore mathematics curriculum (Ministry of Education, Singapore, 2006, p. 2)

While problem solving has been a focus of the curriculum since 1992, Kaur and Yeap (2009) report limited implementation in classrooms with textbooks typically containing closed, routine problems and instruction in mathematics lessons usually teacher-led. In response to the limited implementation of problem solving by teachers, examinations have recently contained novel, non-routine problems. Teachers are now being confronted with new challenges to design and use similar tasks in their lessons. In addition to this, two new initiatives *Thinking School, Learning Nation* (TSLN) and *Teach Less, Learn More* (TLLM) have aimed to reduce the curriculum content further and engage students in more thinking and problem-solving tasks (Kaur & Yeap, 2009). As evidence of the government's commitment to teachers and their growth as professionals, teachers are entitled to 100 hours of professional development every year (Kaur, 2001).

Hong Kong

In his presentation at a forum organised by the National Curriculum Board Wardlaw (2008) revealed Hong Kong has undergone significant reform since 2000 with a focus on student learning through alignment of curriculum, pedagogy and assessment (Figure 2). Associated with this reform are the following fundamental principles:

- all students have opportunities to learn and should not be screened out early;
- life-long learning capabilities are needed for a contemporary and future world;
- whole person development for enhancing quality of life in society, culture, economy;
- conceptions of knowledge changing – cross disciplinary, personal, co-constructed; and
- structural changes to facilitate opportunities and pathways for all young people (Wardlaw, 2008, slide 5).

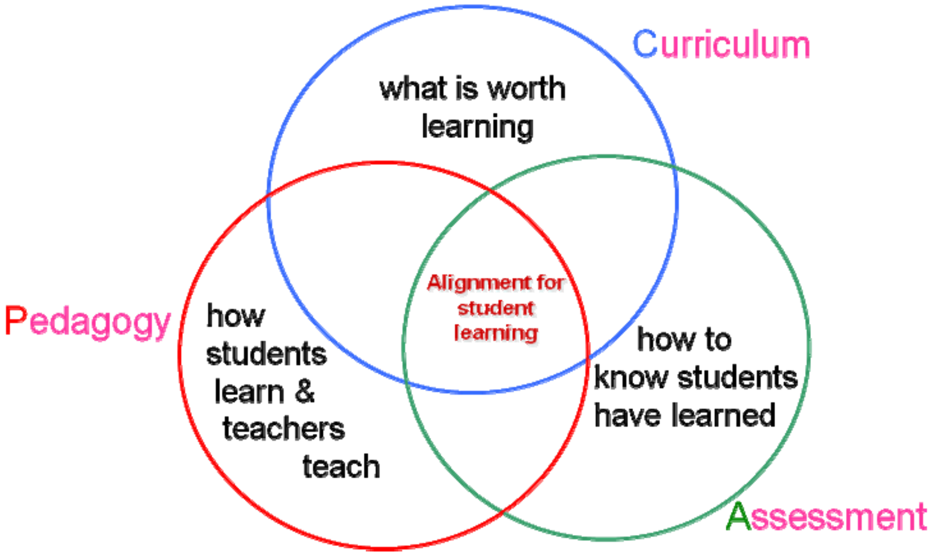

Figure 3. Aligning curriculum, pedagogy and assessment in Hong Kong (Wardlaw, 2008)

The Hong Kong Curriculum Framework has three interconnected components: Key Learning Areas, Generic Skills and Values and Attitudes. Mathematics is one of the Key Learning Areas and the Generic Skills include: collaboration, communication, creativity, critical thinking, information technology, numeracy, problem solving, self-management and study skills. Interestingly, the *Basic Education Curriculum Guide* (Education Dept. HKSAR, 2002) indicates the priority for 2001-2006 was communication, critical thinking and creativity. While Hong Kong has a coherent curriculum with high expectations, which values learning and training in basic skills and fundamental concepts, and with teachers who have good pedagogical content knowledge, Wardlaw (2008) acknowledges students have low self-efficacy and poor attitudes, particularly in mathematics. Additionally, there is an examination orientation, the mathematics curriculum is dense and compact, and the teaching and learning is rushed.

Teachers in Hong Kong are more aware of problem-solving approaches to teaching mathematics, but there remains limited evidence of implementation. For those teachers who try to engage students in discussion, mathematical reasoning and problem solving, they continue to lead students on a predetermined solution pathway rather than allowing more open investigation and exploration of mathematical ideas (Mok, Cai & Fung, 2005). Observations in Year 1 classrooms were characterised by "whole-class teacher-pupils interaction and highly structured group/pair work" (Mok & Morris, 2001). More recently, Mok and Lopez-Real (2006) noted little use of group work or open-ended questions suitable for exploratory problem solving in the lessons of Hong Kong secondary school teachers.

England

The latest mathematics curriculum documents in England for Key Stage 3 and Key Stage 4 (the first four years of secondary education) are less prescriptive allowing more flexibility for teachers. They contain a framework of personal learning and thinking skills and have a focus on assessment *for* learning. Problem solving is described as "lying at the heart of mathematics" (DCSF, 2008a, p. 5) and is represented as a cycle of processes including representing, analysing, interpreting and evaluation, and communicating and reflecting. The explanation for the relationships depicted in Figure 4 is "the diagram represents the dual nature of mathematics: it is both a tool for solving problems in a wide range of contexts and a discipline with a distinctive and rigorous structure" (DCSF, 2008a, p. 19).

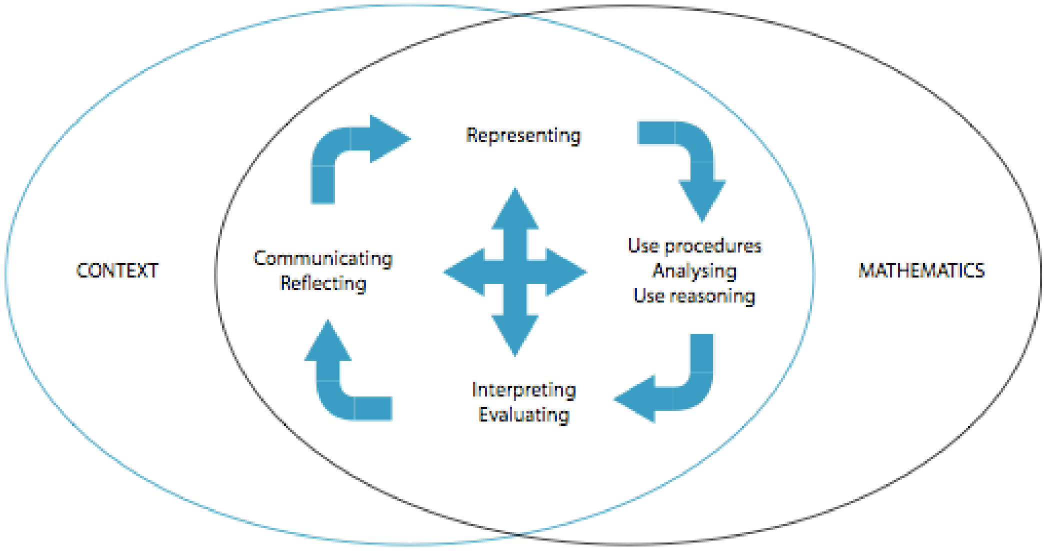

Figure 4. A representation of the processes involved in problem solving (DCSF, 2008a, p. 6).

To assist teachers, a wide range of support material has been prepared for school and district-based professional development with examples of problems and rich tasks for each of the content strands. Teachers are encouraged to analyse tasks to identify the processes, e.g., a task involving finding patterns and relationships in a hundreds chart is accompanied by the template presented in Figure 5. This support is vital if teachers are to embed the processes in lessons and provide regular problem-solving opportunities for students. However, it is too soon to determine the impact of the changes but assessment items will also be changed to include more open-ended questions.

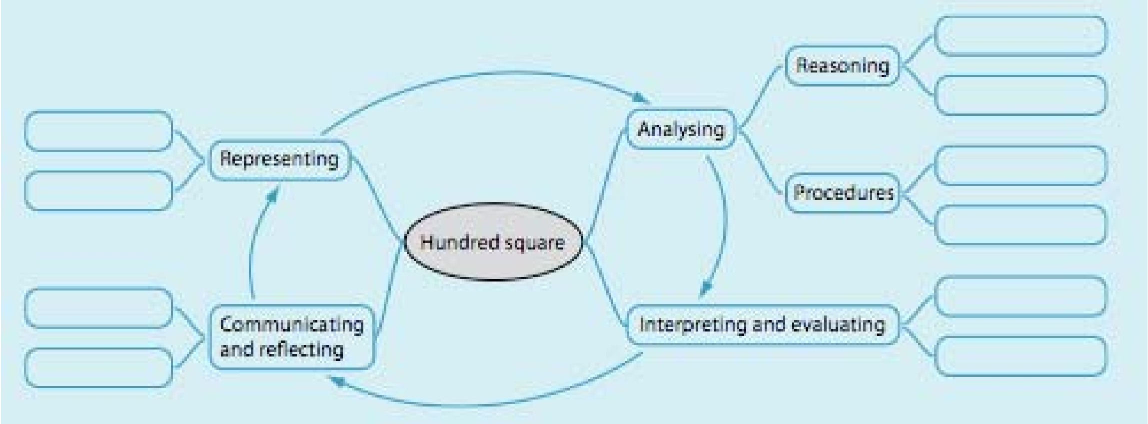

Figure 5. Template to aid teacher identification of processes in a task involving a hundreds chart (DCSF, 2008b, p. 4).

The Netherlands

For at least 30 year, researchers from the Freudenthal Institute in the Netherlands have been developing a mathematics curriculum and a pedagogical approach known as Realistic Mathematics Education (RME). The framework is based on the notion that mathematics is a human activity and that students need to experience 're-inventing' the mathematics for themselves or 'mathematizing' during lessons. Problems based on *imaginable* contexts (those which make sense to students) are used to develop mathematical skills and processes. Rather than using a more traditional teaching approach of demonstration of formal mathematics followed by skills practice and then applications to problems, this approach uses *realistic* problems as a starting point for learning and applying new mathematical ideas. However, for some students a more formal problem may be appropriate since the focus is on problem contexts that are 'imaginable' or 'realisable' for the learner (Van den Heuvel-Panhuizen, 2003). The theoretical approach developed in the Netherlands has been adapted in several other countries including the United States and England (see for example Romberg, 2001).

Teachers have freedom in determining the curriculum although textbooks represent the main source of guidance followed by Key Goals and domain descriptions provided by the Dutch government – interestingly, 'problem solving' is not listed explicitly as one of the goals. Given this flexibility, what is taught in most schools is very similar (Van den Heuvel-Panhuizen, 2000). More recently learning trajectories for particular content topics have been developed to assist teachers but they are not meant as a 'recipe' for what and how to teach. While RME aimed to support the implementation of a problem-oriented curriculum, there is little evidence of non-routine problem solving in Dutch classrooms (Doorman et al., 2007). A lack of such problems in both textbooks and examinations is cited as the main reason for limited implementation.

Considering the mathematics curriculum and problem solving initiatives of Singapore, Hong Kong, England and The Netherlands reveals some similarities and some differences. Singapore made a significant change by reducing mathematics content, the RME approach in the Netherlands was designed to build mathematics learning from relevant problem contexts, and in England the latest curriculum provides increased flexibility and examples of rich problem-solving tasks. Hong Kong is yet to develop the same levels of support for teachers. All recognise that for teachers to include more problem solving opportunities in lessons, textbooks will need to include more examples of problems and examinations need to assess problem solving.

The New Australian Curriculum Approach

The *Australian Curriculum: Mathematics* is currently being written with opportunities for consultation early in 2010 and implementation in 2011. The guiding statement for writers, the *Shape of the Australian Curriculum: Mathematics* (National Curriculum Board [NCB], 2009), presents the structure as three content strands – Number and algebra, Measurement and geometry, and Statistics and probability – as well as four proficiency strands – understanding, fluency, problem solving and reasoning (informed by Kilpatrick, Swafford & Findell, 2001). Problem solving is described as "the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively" (NCB, 2009, p. 6). Expectations for problem solving will be elaborated to support teaching and assessment – this is critical since teachers will need models of practice to support effective implementation.

Kilpatrick et al. (2001, p. 5) included a fifth proficiency, *productive disposition*, described as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy". It is disappointing that this proficiency was not included in the *Mathematics Shape Paper* since it is critical that we focus on developing positive dispositions in mathematics, particularly given recent reports highlighting negative views about both the content and teaching of mathematics, particularly in the early secondary years (McPhan, et al., 2008). The decision to exclude this proficiency was probably based on the need to develop achievement standards which would be difficult to write. However as Kilpatrick et al. (2001) note, the five proficiencies are "interwoven and interdependent" which will make the writing of separate achievement standards for each of the proficiencies a challenging task.

Concluding Comments

In a summary of international trends in mathematics curriculum development, Wu and Zhang (2006) noted an increased focus on problem solving and mathematical modelling in countries from the West as well as the East. Curriculum developers recognise that providing problem-solving experiences is critical if students are to be able to use and apply mathematical knowledge in meaningful ways. It is through problem solving that students develop deeper understanding of mathematical ideas, become more engaged and enthused in lessons, and appreciate the relevance and usefulness of mathematics.

Given the efforts to date by many countries (including Australia) to include problem solving as an integral component of the mathematics curriculum and the limited implementation in classrooms, it will take more than rhetoric to achieve this goal. While providing valuable resources and more time are important steps, it is possible that problem solving in the mathematics curriculum will only become valued when it is included in high-stakes assessment. In addition, teachers need readily available examples of useful non-routine problems, particularly in textbooks.

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problems; and (iv) the role played by the use of both multiple purpose and ad hoc mathematical action types of technologies in problem solving activitiesâ€"what ways of reasoning do learners construct when they rely on the use of digital technologies and how technology and technology approaches can be reconciled?Â It has infused mathematics curricula around the world with calls for the teaching of problem solving as well as the teaching of mathematics through problem solving. And as such, it has been of interest to mathematics education researchers for as long as our field has existed.Â Mathematical problem-solving competences are thus acquired through the promotion of manifestations of mental agility (reduction, reversibility, minding of aspects and change of aspects). Problem-solving has played a central role in the thinking of educational theorists ever since the publication of PÃ³lyaâ€™s book â€œHow to Solve It,â€ in 1945. The National Council of Teachers of Mathematics (NCTM) has been consistently advocating for problem-solving for nearly 40 years, while international trends in mathematics teaching have shown an increased focus on problem-solving and mathematical modeling beginning in the early 1990s.Â Given the efforts to date to include problem-solving as an integral component of the mathematics curriculum and the limited implementation in classrooms, it will take more than rhetoric to achieve this goal. The present Mathematics curriculum at the upper primary level aims to develop a number of mathematical skills and processes among children in classes VI-VIII. Curriculum for classes VI to VIII is designed to ensure that children build a solid foundation in mathematics by connecting with mathematical concepts learnt in primary classes and applying them in a variety of ways and situations.Â What changes are envisaged in the role of the teacher? 11. Unfolding the Curriculum: Mathematics Curriculum in Practice. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable. mathematics 2 curriculum guide 2017. 1.Â There are a number of components that define the nature of mathematics and these are woven throughout this curriculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty. It is important for students to understand that mathematics is dynamic and not static.